HW4 – Lambdas

Deliver your work on paper by 13:15 on **March 2**.
Don't forget to put your name and studentnumber on it.
Email submissions and late submissions (after 13:15 on March 2) will not be accepted.

1.
Give informal descriptions of the following functions, which are described as lambda terms. For each function give the type of the function, and then describe the function itself as well as its un-Curried version.
For example, $\lambda x_e.\lambda y_e.y$ is a function of type $e(ee)$. It sends every entity of type e to the identity function of type ee, i.e. to the function that sends every entity of type e to itself. Equivalently, $\lambda x_e.\lambda y_e.y$ is the Curried version of the function that takes two entities of type e and returns the second one as its result.

1. $\lambda x_e.\lambda f_{et}.f(x)$
2. $\lambda f_{(et)t}.\lambda y_t.f(\lambda z_e.y)$
3. $\lambda f_{ee}.\lambda x_e.f(f(x))$
4. $\lambda f_{ee}.\lambda g_{(ee)t}.\lambda x_e.g(\lambda y_e.f(x))$
5. $\lambda f_{e(et)}.\lambda x_e.\lambda y_e.(f(x))(y) \wedge (f(y))(x)$
   remark: $\wedge$ is standard propositional conjunction.

2. Write down the lambda terms corresponding to the following informal descriptions.

1. The function of type $(e(ee))(((ee)t)(et))$ that sends two functions of types $e(ee)$ and $(ee)t$ to their composition. Recall that the composition of a function $f : A \to B$ and a function $g : B \to C$ is the function $h : A \to C$ s.t. for every $x \in A$, $h(x) = g(f(x))$.
2. The function of type $(e(et))(e(et))$ that sends every (char. function of a) binary relation R to the (char. function of the) symmetric closure of R. Recall that R' is the symmetric closure of R if for all pairs $\langle x, y\rangle$ in R, R' contains both $\langle x, y\rangle$ and $\langle y, x\rangle$.
3. The function of type $((ee)e)e$ that sends every function of type $(ee)e$ to its value on the identity function of type ee.

3. Consider the following equivalence: *Tina is a tall person* $\Leftrightarrow$ *Tina is tall and Tina is a person*.
We assume that the words *is* and *a* both denote the identity function of type $(et)(et)$. We also assume the binary structures below:

Tina [is [a [tall person]]]

[Tina [is tall]] [and [Tina [is [a person]]]]

Let us assume that the type of the word *tall* in the constituent *tall person* is of type $(et)(et)$, and thus different than the et type of the word *tall* in the constituent *is tall*. Think of a proper denotation of type $(et)(et)$ for the word *tall* that would respect the equivalence above.

1. Describe this $(et)(et)$ function informally.
2. Write it as a lambda term.
3. Write down all the other lambda terms for the words in the two sentences of the equivalence.
4. Using the two binary structures above, and the lambda terms for all the words in them, write down lambda terms for the two sentences.
5. Simplify each of these lambda terms, and verify that you reached the same term.