

Quiz in Set Theory

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Instructions

1. The quiz contains 5 questions. Please answer all of them on the exam sheet.
2. You are not permitted to use any help from people or pre-prepared material.
3. Exam duration: 30 minutes.
4. Please write your student number here: _____.

Good luck!

Question 1 (12 points)

Define all the functions in the set $\{a, b\}^{\{1,2\}}$.

Question 2 (12 points)

Write down all the members in the set $\wp(\{a, b, c\}) \cap \wp(\{b, c, d\})$.

Question 3 (21 points)

Which statements below are false, if any?

- a. $\emptyset \in \wp(\wp(\emptyset))$
- b. $\emptyset \subseteq \wp(\wp(\emptyset))$

- c. $\{\emptyset\} \subseteq \wp(\wp(\emptyset))$
- d. $\{\emptyset\} \in \wp(\wp(\emptyset))$
- e. $\{\{\emptyset\}\} \in \wp(\wp(\emptyset))$
- f. $\{\{\emptyset\}\} \subseteq \wp(\wp(\emptyset))$

Question 4 (30 points)

We are given two sets A, B s.t. $A \neq \emptyset, B \neq \emptyset$ and $A \neq B$, and two functions:

$$f : (A \times B) \rightarrow A \quad \text{s.t. } \forall x \in A \forall y \in B: f(\langle x, y \rangle) = x.$$

$$g : (A \times B) \rightarrow (B \times A) \quad \text{s.t. } \forall x \in A \forall y \in B: g(\langle x, y \rangle) = \langle y, x \rangle.$$

For each of these two functions, complete the following table with “yes” and “no”:

	injection?	surjection?	bijection?
f			
g			

Choose the right possibility in each of the following two questions, and fill in the missing data, if needed:

1. The function f^{-1} is:

- a. undefined
- b. defined as the function in the set _____ s.t f^{-1} satisfies:
_____.

2. The function g^{-1} is:

- a. undefined
- b. defined as the function in the set _____ s.t g^{-1} satisfies:
_____.

Question 5 (25 points)

Let f and g be the two functions in the set $\{1, 2, 3\}^{\{1,2,3\}}$ that satisfy:

$$f(1) = 2, f(2) = 2, f(3) = 3.$$

$$g(1) = 1, g(2) = 1, g(3) = 3.$$

Answer the following questions:

1. When looking at f and g as binary relations, consider the relation $f \cup g$. Answer the following questions.
 - a. Is $f \cup g$ symmetric? yes/no
 - b. Is $f \cup g$ reflexive? yes/no
 - c. Is $f \cup g$ transitive? yes/no
2. The function $g \circ f$ is the function in the set _____ s.t. $g \circ f$ satisfies:
_____.