Semantiek HW1: Set Theory + Reading (8 February 2012)

Definitions of operators and properties can be found in Iemhoff's notes, see the link on the website.
If you have to prove something: explain your steps formally without using informal arguments (e.g. Venn diagrams or any other metaphor).
Please submit your work **on paper** *in the mailbox (Janskerkhof 13a, in de koffiecorner staat de kast) not later than* **February 15 at 13:15**.
Don't forget to put your name and studentnumber on it!
We will not accept late submissions or submissions sent by email.
Please pay special attention to all details*: make sure everything is clear to you and consult teacher on WC (February 10) if in doubt. The material in this assignment is very basic: its mastery is a strong prerequisite for following this course.*

Preparation

- For the meeting of February 15, read Partee's *Reflections* (link on website). You can also start reading Abbott's paper (link on website).
- Make sure you master all the following notions (from *Wiskunde Voor AI 2*):
  - element, gelijkheid, deelverzameling
  - lege set and set constructie
  - set vereniging, doorsnede, complement en verschil
  - machtsverzamelingen
  - geordende paren, cartesische producten
  - Relaties, domein, bereik
  - Eigenschappen van relaties: symmetrie, transitiviteit, reflexiviteit, etc.
  - functies
  - inverse functies, functiecompositie
  - injectie, surjectie, bijectie
  - isomorfisme

If you are not completely sure about the exact definition (or use) of each of these notions, please consult Iemhoff's notes (link at the course website).

Exercises:

1. Let E be a set and a, b and c be members of E.
   a Is $\{a, b, c\} \in \{\{a\}, \{b\}, \{c\}\}$?
   b Is $\{a, b, c\} \subseteq \{\{a\}, \{b\}, \{c\}\}$?
   c Is $\{a, b, c\} \subseteq \bigcup\{\{a\}, \{b\}, \{c\}\}$?
   Remark: if $\mathcal{A}$ is a set of sets, then by $\bigcup \mathcal{A}$ we mean the union of all the sets A that satisfy $A \in \mathcal{A}$.
   d Is $\{\{a, b, c\}\} \subseteq \wp(E)$?
   e Is $\{\{a, b, c\}\} \in \wp(E)$?
   f Is $\emptyset \in \{\{a\}, \{b\}, \{c\}\}$?
   g Is $\emptyset \subseteq \{\{a\}, \{b\}, \{c\}\}$?
2. a Laten we aanemen dat $\langle a, b\rangle$ als {a, b} gedefineerd is. Bewijs dat de volgende stelling niet geldt: $\langle a, b\rangle = \langle c, d\rangle$ desda $a = c$ and $b = d$.
   b Laten we nu de standaard definitie gebruiken: $\langle a, b\rangle$ = {{a}, {a,b}}. Bewijs dat nu de stelling wel geldt.
3. Let E be a set. Let $A, B \subseteq E$ s.t. $A \neq \emptyset$ and $B \neq \emptyset$. Prove that the cartesian product $A \times B$ is a binary relation over E that is symmetric if and only if $A = B$.
   Tip: for the "only if" direction, you can assume for contradiction that A is not equal to B, and then show that one of the following must hold: $A \setminus B \neq \emptyset$, $B \setminus A \neq \emptyset$. And then you can use this fact to find a pair in $A \times B$ that is not in $B \times A$, or vice versa.
4. Show that given a reflexive relation R, the relation S defined by

$$Sxy \Leftrightarrow Rxy \vee Ryx$$

   is a reflexive symmetric relation.
5. Laat a, b en c drie verschillende elementen van een verzameling E zijn. Laat zien dat het aantal functies van $\{a, b, c\}$ naar $\{0, 1\}$, m.a.w de omvang van $\{0, 1\}^{\{a,b,c\}}$, 8 is (dat wil zeggen 2^3), door alle functies expliciet als sets te geven.
6. Show that for a finite set A there is no surjection from A to $\mathcal{P}(A)$.