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Karttunen logic for presupposition projection[1]

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Abstract. Presuppositions of complex sentences are empirically distinguished from the propositional contexts that render a sentence coherent. This distinction is at the heart of the *proviso problem* for presupposition projection. Here we show that Karttunen's *inference-based* approach in his proposals from the early 1970s can be used to directly avoid the proviso problem. Inference-based projection is different from trivalent accounts or satisfaction-based methods in distinguishing presuppositions from admittance conditions on contexts. This distinction is used within a new propositional fragment, whose rules for updating local contexts and satisfying presuppositions are explained using the same Incrementality principle of previous accounts, but without any of the additional assumptions that have been used to tackle the proviso problem.

Keywords: presupposition, admittance, proviso problem, Karttunen Logic.

1. Introduction

Two empirical notions of presupposition are prevalent in the literature. Classically, presuppositions are characterized as conclusions that follow from a sentence and its negation, as well as questions, conditionals and modal compounds that contain it (Chierchia and McConnel-Ginet, 1990). Another approach focuses on necessary admittance conditions on sentence coherence. For example, the sentence *Sue's aunt has arrived* does not only entail that Sue has an aunt, but also requires that Sue has an aunt for being a communicative speech act (Stalnaker, 1978).

As far as simple sentences like that are concerned, it does not seem particulary important if we characterize presuppositions as entailments or as admittance conditions. However, complex sentences often show a discrepancy between the two phenomena, as in the following example:

(1) If Myanmar is an autocracy, its king will be able to overcome the current crisis.

Without further assumptions, from (1) on its own we must draw the (factually incorrect) conclusion that Myanmar is a monarchy. Accordingly, this conclusion is classified as a presupposition of (1). However, for a hearer who assumes that all autocracies have a king, (1) might be coherent and meaningful, without the conclusion that Myanmar has a king. This kind of contrasts challenges theories that conflate presuppositions and admittance conditions, and it is the essence of the familiar *proviso problem* for such theories (Geurts, 1996).

A common approach to the problem has been to formally derive admittance conditions, and rely on additional principles to strengthen them into presuppositional entailments. This approach contrasts with the proposals in (Karttunen, 1973, 1974), which derive presuppositions relative to contexts, and define admittance conditions as those contexts that render a sentence presupposition-less. For sentences like (1), Karttunen derives different presuppositions in different contexts. In the realistic context where no inferential relation is assumed between autocracies and kings, (1) presupposes that Myanmar has a king. However, a context where all autocracies have a king makes (1) presupposition-less. This distinction employs inferential relations between syntactic forms as the key for presupposition "filtering", or *satisfaction*.

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Despite the big influence of (Karttunen, 1973, 1974), inferential mechanisms of presupposition satisfaction are not very popular, and have only been partly revisited in recent work (Mandelkern, 2016a; Francez, 2019; Winter, 2019). This paper develops this line by focusing on the empirical differences between Karttunen's approach and more recent treatments of presupposition. The most notable advantage of Karttunen's proposals is that they do not face the proviso problem. This advantage is discussed in section 2, also in relation to the so-called *conditional presuppositions*. Karttunen's (1974) analysis shows an empirical advantage over (Karttunen, 1973), which relies on representing the *local context* of each presupposition trigger. The connection between this treatment and alternative approaches is discussed in section 3. In section 4 it leads us to generalize Karttunen's rule into a fully recursive propositional calculus, called *Karttunen Logic*. Despite its representational nature, Karttunen logic is rooted in the incremental reasoning behind the asymmetric Kleene truth tables (Schlenker, 2008a; Fox, 2008), which does not require any *ad hoc* assumptions on representational manipulations.

2. Karttunen's inferential approach and the proviso problem

Karttunen (1973, 1974) – henceforth *K73,K74* – introduced some of the key observations and concepts that shaped our current understanding of presuppositions. Both proposals crucially rely on inferential relations between syntactic objects. In Karttunen's analysis, whether a presupposition is projected or inhibited depends on whether it is logically entailed by its trigger's environment. Karttunen argues that this avoids undesirable predictions of trivalent analyses.[2] The same challenge was later identified by Geurts (1996) for possible world accounts of presupposition satisfaction (Heim, 1983; Stalnaker, 1978). Geurts referred to this challenge as the *proviso problem*. The fact that possible world accounts suffer from the same problem as trivalent analyses is logically unsurprising: both approaches rely on situation-specific denotations as the key for presupposition projection, and their canonical versions are known to be equivalent (Peters, 1979; Winter, 2020). As Francez (2019) observes, these denotational mechanisms "obliterate Karttunen's distinction between the presuppositions of an expression and what it takes for a context to satisfy them". The present section discusses the proviso problem as an artefact of that obliteration, contrasting it with Karttunen's inferential approach.

2.1. Presuppositions and admittance conditions

When sentence (2) below is interpreted out of the blue, language-speakers readily infer from it that Dan has a beard:[3]

(2) If Sue visits Dan, she will like his beard.

We classify this inference as a ***presupposition*** of the description *Dan's beard*. More generally, presuppositions are characterized as a subspecies of entailment that, unlike other entailments, are preserved under negation, questions, conditionals and various intensional expressions (Chierchia and McConnel-Ginet, 1990; Beaver and Geurts, 2014).

[2] K73 (p.188) notes that Strong Kleene accounts of presuppositions counter-intuitively expect the sentence *Marseilles is the capital of France, and the king of France is bald* not to presuppose that there is king of France. This happens whenever the first conjunct in the sentence is false, as in the current state of affairs.

[3] Here and henceforth I ignore tense issues, leaving the possibility open that some speakers only conclude from (2) that Dan will have a beard, and not necessarily that he already has one. I also discuss conditionals as if they were material implications, as this (unrealistic) assumption hardly affects the analysis of presupposition projection.

The empirical study of presuppositions is complicated by the effects of contextual information. As Karttunen observed, what a complex sentence like (2) presupposes may be affected by the context in which it is uttered. For example, let us consider the following scenario:

(3) Dan has no beard, and he is not particularly fond of beards, but he knows that Sue likes them. Intending to impress Sue, Dan decides that if she visits him, he'll grow a beard before her arrival. If Sue does not visit him, he will stay well-shaven.

Sentence (2) is coherent in context (3), but now we no longer infer from it that Dan has a beard. We say that context (3) *admits* (2), thereby inhibiting its out-of-the-blue presupposition. Intuitively, context (3) establishes the causal connection expressed by the following conditional:

(4) If Sue visits Dan, he will have a beard.

Having (4) as part of our background assumptions is sufficient for using sentence (2) coherently. We call (4) an ***admittance condition*** of (2): a logically weakest context that admits the sentence. In any context like (3) that entails (4), we do not conclude from (2) that Dan has a beard. This observation is surprising: how can the conditional (4) inhibit a presupposition ("Dan has a beard") that it does not logically entail?[4] Despite many works on the proviso problem, this puzzle has remained one of the greatest challenges for theories of presupposition.

2.2. Conditional analyses vs. Karttunen's analysis

Many previous accounts formally derive conditionals like (4) as part their presupposition projection mechanism. Mandelkern (2016b) refers to it as the *C-analysis*. In this analysis, the statement that a speaker is invited to accept when hearing a sentence like (2) is not its traditional presupposition ("Dan has a beard") but the weaker admittance condition (4).[5] The "proviso problem" for this approach is to explain how in out-of-the-blue contexts, speakers' conclusion from (2) is the stronger classical presupposition: that Dan has a beard. To address this problem, C-analyses rely on additional strategies that aim to strengthen admittance conditions into the observed presuppositional inferences. This approach was initiated by Karttunen and Peters (1979) and Heim (1983), and was followed, using different mechanisms by van Rooij (2007), Schlenker (2011) and Lassiter (2012), among others.

Karttunnen (1973,1974) presented a different approach, which relies on logical entailment. According to Karttunen, whether a presupposition is projected or inhibited depends on whether it is entailed by the syntactic environment of its trigger. In sentence (2), the trigger *his beard* presupposes that Dan has a beard. The antecedent *Sue visits Dan* does not on its own entail that presupposition. Thus, when (2) is used in an out-of-the-blue context, the presupposition is not locally satisfied, and is inherited by the matrix sentence (2). By contrast, when a speaker or a hearer finds herself in a context where she accepts (4), this background together with the antecedent *Sue visits Dan* entails that Dan has a beard. In such contexts the presupposition of the consequent is locally satisfied. Consequently, sentence (2) is admitted and is rendered

[4]That is to say, presuppositional reasoning can be non-monotonic: (2) presupposes, hence entails, that Dan has a beard, but the conjunction of (4) and (2) does not entail that. By contrast, classical reasoning is monotonic: if a proposition p classically entails q, then $C \wedge p$ also entails q for any proposition C.

[5]Some C-analyses refer to conditionals like (4) as *presuppositions* (Heim 1983: p.252; Beaver 2001: p.82). This technical label should not be confused with the traditional empirical characterization of presuppositions using their special inferential properties ("projection"). See more on this point in section 3.3.

presupposition-less. I refer to this inferential approach as the *K-analysis*.[6]

2.3. Problems for the C-analysis

As we saw, C-analyses rely on conditionals like (4) to be strengthened into the actual conclusion *Dan has a beard* that speakers draw from sentence (2) in out-of-the-blue contexts. One problem with this approach was pointed out by Geurts (1996) and involves examples like the following:

(5) Sue and Dan's friends know that if she visits him, he will have a beard.

In (5), the factive verb *know* triggers the conclusion *if Sue visits Dan, he will have a beard*. In out-of-the-blue contexts, the K-analysis correctly expects this conditional conclusion as a presupposition of (5). The C-analysis derives the same conditional as an admittance condition of (5). Thus, if we follow the C-analysis of sentence (2), we may expect this admittance condition of (5) to be strengthened. As a result, without further restrictions on strengthening the C-analysis derives for (5) a presupposition that is unnecessarily strong. Another problem with the C-analysis is pointed out by Mandelkern (2016a, b), who notes many cases where presuppositions are stronger than what C-analyses expect.

Here I would like to point out another problem for the C-analysis: in some cases, even the weak conditional presuppositions that it derives are too strong. To see that, let us first consider the influence of implicit contextual information in the following, non-problematic, example:

(6) If Sue is married, she brought her spouse to the party.

Given common knowledge on marriage, the conditional *if Sue is married, she has a spouse* must be trivially true in any reasonable context. Both the C-analysis and the K-analysis expect this conditional to be an admittance condition of (6). Thus, both analyses explain why when (6) is uttered out of the blue, it is admitted without the presupposition that Sue has a spouse.

In the case of sentence (6) there is little doubt about the relevant common knowledge. However, much of the discussion surrounding C-analyses concerns examples where common knowledge is not that clear. A typical case is another example by Geurts (1996):

(7) If Theo is a scuba diver, then he will bring his wet suit.

Intuitively, sentence (7) does not presuppose that Theo has a wet suit. In this sense sentence (7) is similar to (6). However, in sentence (7) it is much less clear than in (6) what an out-of-the-blue context might consist in. While the assumption *all married people have spouses* is reasonably shared by all English speakers, the statement *all scuba divers have wet suits* is less likely to be universally accepted. Thus, unlike sentence (6), it is not guaranteed that an out-of-the-blue utterance of sentence (7) should have its admittance condition satisfied.

According to the C-analysis, sentence (7) has the conditional presupposition *if Theo is a diver he has a wet suit*. The hearer is supposed to infer this conclusion as part of presupposition accommodation, without any application of strengthening. In the K-analysis, the presupposition that is derived for (7) depends on what we assume the context to be. If we can infer from the context that *if Theo is a diver he has a wet suit*, sentence (7) is admitted. If we can not, the

[6] K73 focused on presuppositions in the classical sense above, whereas K74 focused on presupposition satisfaction. However, in relation to the proviso problem we do not need to distinguish the two approaches (see section 3.3).

presupposition is *Theo has a wet suit*. When elaborating their K-analyses, Mandelkern (2016a) and Winter (2019) point out that a conditional like *if Theo is a diver he has a wet suit* may be inferred by a hearer even when the context does not logically entail it. In the case of (7), out-of-the-blue contexts are expected to support the generic statement "scuba divers have wet suits". The conditional *if Theo is a diver he has a wet suit* is not a logical conclusion from this generic, but it is a default conclusion (Pelletier and Asher, 1997). According to Mandelkern and Winter's K-analyses, this default is responsible for the fact that speakers may accept sentence (7) in out-of-the-blue contexts without inferring from it that Theo has a wet suit.

In relation to (7), the differences between the C-analysis and the K-analysis are mostly theoretical. However, it is also instructive to look at cases where the background knowledge is less fuzzy than with scuba divers and their potential wet suits, as in the following example:

(8) When shooting a nature film in an African savanna, a director and a cameraman notice a big animal hiding in the grass. The director tells the cameraman:
If this animal is a lion, you should focus on its mane.

The cameraman would probably not infer from (8) that if the animal in the grass is *any* lion, it has a mane. This would be odd, given that the director clearly doesn't know which animal it is, let alone a male lion. Still, the director's utterance is fully intelligible. To analyze (8), it is natural to assume that the generic *lions have manes* is in the background. Since the director and cameraman are likely to know that male lions have manes but females do not, their generic knowledge quietly triggers restricting the domain of lions in (8) to male lions. Such a domain restriction renders (8) coherent independently of the analysis of presupposition projection, and it also accounts for sentences like (7) without conditional presuppositions.

2.4. Summary: the promise of the K-analysis

The C-analysis and the K-analysis differ in the role they assign to pragmatic processes. In the C-analysis, the derivation of conditional presuppositions is not the end of the game. Additional conversational considerations or semantic processes must apply in order to strengthen the derived presupposition and avoid the proviso problem. Despite efforts to analyze these processes, there is still much controversy surrounding them. By contrast, in the K-analysis, as soon as the context is specified, presupposition projection is fully predictable. This advantage calls for a more systematic study of Karttunen's inferential hypothesis on presupposition projection.

3. Inferential incrementality and local contexts

While Karttunen's inferential approach is not threatened by the proviso problem, there are some other immediate questions that it must address. This section first addresses a question about its explanatory power. Recapitulating the proposal in (Winter, 2019), it is shown that K73's rules are derived from the same incremental principles that have been used to motivate denotational trivalent accounts. We then turn to examples where presuppositions are inhibited due to information that is accumulated from different clauses. K74's notion of *local context* accounts for these effects. We show that the principles that govern updates of local contexts are the same incremental principles that explain K73's rules. It is concluded that trivalent accounts and the proposals by K73 and K74 have a common logical backbone, which is further developed in the propositional *Karttunen Logic* of section 4.

3.1. Presupposition projection – the explanatory challenge

A central criterion for evaluating theories of presuppositions is their ability to predict presupposition projection on the basis of the syntax and classical semantics of various operators (Schlenker, 2008b). This explanatory puzzle is manifest when we compare the behavior of propositional connectives. Let us reconsider the conditional sentence (6), restated in (9a) below with the conjunctive and disjunctive variations in (9b-c):

(9) a. *If* Sue is married, she brought her spouse to the party.
b. Sue is married *and* she brought her spouse to the party.
c. Sue is unmarried *or else* she brought her spouse to the party.

In (9a) and (9b), the presupposition about Sue's spouse is inhibited by the first operand, which asserts that Sue is married. A similar inhibition occurs in (9c), where the first operand asserts that Sue is *un*married. The explanatory challenge here is to account for the similarity between conditionals and conjunctions, and their difference from disjunctions. The trivalent truth tables in (Peters, 1979) elegantly address this challenge. As pointed out in (Fox, 2008; Schlenker, 2008b; George, 2014), Peters's tables are derived by an incremental approach to presupposition projection. Binary constructions such as (9a-c) are interpreted by first interpreting one of the operands, which in (9a-c) is the one on the left.[7] If the truth-value of this operand is sufficient to determine the result of the operation, the other operand is ignored. In material conditionals and conjunctions, falsity of the first operand determines the result (as *true/false*, respectively). In (9a) and (9b), this happens when Sue is unmarried. Therefore, we only need to consider the presupposition of the second operand if Sue is married, but such situations render the presupposition about Sue's spouse true. Consequently, (9a-b) are correctly expected to show no presuppositional effect. With disjunctions as in (9c), it is truth of the first operand that determines the result. Thus, according to the same incremental analysis, the second disjunct in (9c) is ignored when Sue is unmarried. The net effect is again presupposition inhibition.

To summarize, the following principle accounts for Peters's tables for any construction $\alpha \,\mathsf{op}\, \beta_{\mathsf{p}}$ where p is the presupposition of the second operand:

(10) **Denotational Incrementality**: In a binary construction $\alpha \,\mathsf{op}\, \beta_{\mathsf{p}}$, the presupposition p is *inhibited by a situation S* if α's denotation in *S* determines the result of the operation.

This principle is called "denotational" because whether the presupposition p is inhibited only depends on the truth-value of the first operand in the given situation S. This leads to the C-analysis from section 2: an analysis that generates conditional presuppositions, and hence is sensitive to the proviso problem. To see that, let us consider the following variations on (9a-c):

(11) a. *If* Sue is happy, she brought her spouse to the party.
b. Sue is happy *and* she brought her spouse to the party.
c. Sue is unhappy *or else* she brought her spouse to the party.

In these sentences, any situation where Sue is unhappy makes the first operand determine the result. Thus, using denotational incrementality, we expect the presupposition about Sue's spouse to be inhibited in any situation where she happens to unhappy. This is an instance of the proviso

[7]There are also binary constructions where the first interpreted operand is the one that linearly appears on the right. See (Mandelkern and Romoli, 2017) on antecedent-final conditionals. Concurrent interpretation of the operands may account for symmetric filtering effects with disjunction (Geurts, 1996).

problem. Equivalently, we may describe this problem by noting that sentences (11a-c) are all counter-intuitively expected to presuppose that *Sue is either unhappy or married*, or, in terms of material implication: *if Sue is happy, she's married.*

In (Winter, 2019) I pointed out that while K73's proposal avoids the proviso problem, its reasoning is remarkably similar to the denotational incrementality principle. According to Karttunen, the inhibition of a presupposition p in $\alpha\,\mathsf{op}\,\beta_{\mathsf{p}}$ is never due to an "incidental" truth-value that α has in a specific situation. Rather, presuppositions can only be inhibited on the basis of an inference relation between the propositions α (or its negation) and p. By definition, entailments express relations between denotations of propositions in all relevant situations. Thus, K73's proposal is formally encapsulated by the following principle:

(12) **Inferential Incrementality**: In a binary construction $\alpha\,\mathsf{op}\,\beta_{\mathsf{p}}$, the presupposition p is *inhibited* if every situation S either makes p true or makes α's denotation determine the result of the operation.

Principle (12) specifies a certain inferential relation between the propositions α and p, which is postulated to inhibit p's effect. This relation makes sure that in any situation where p is not simply *true* (hence satisfied), the result of the operation is determined by α alone. In (11a) this relation does not hold, as seen in any situation where Sue is happily unmarried. In such situations, the antecedent ("Sue is happy") is true, hence it does not determine the result of the conditional, but the presupposition "Sue is married" is false. Consequently, principle (12) expects the presupposition about Sue's spouse not to be inhibited in (11a). A similar analysis correctly expects the presupposition projection in sentences (11b) and (11c). Thus, Inferential Incrementality (12) addresses the explanatory challenge using the same reasoning of Denotational Incrementality (10) without leading to the proviso problem.

3.2. Accumulation of local contexts

Inferential Incrementality (12) does not model the contextual effects on presupposition projection that were discussed in section 2. Following K73, we can model these effects by restricting the situations that are quantified over in (12) to contextually relevant situations. While this method works when the contextually assumed facts are implicitly believed by speakers, it misses a generalization about complex constructions. For example, let us consider the sentence that is formed by conjoining sentence (2) with its admittance condition (4):

(13) If Sue visits Dan he will have a beard, *and* [*if* she visits him, she will like his beard].

In (13), the connection between Sue's visit and Dan's beard is explicitly asserted as the first conjunct. The presupposition trigger *his beard* is embedded within the second conjunct. K73's rules and generalization (12) above are stated as if the inhibition of presuppositions is only a matter of simple binary constructions. This does not cover embeddings of binary constructions like (13), or the following similar examples:

(14) *If* Sue is at home, *then* [she is working now <u>and</u> *Dan knows that Sue is at home working*].

(15) Sue is miserably married with miserable kids, *and* [*if* her children ever get married, then *Sue's [miserable children]'s marriages* will surely be as miserable as Sue's].

(16) All of Rowling's Harry Potter books have been best-sellers, *and* [either Rowling published no Harry Potter book in 2002 or *it was* Harry Potter and the Goblet of Fire *that was a Rowling's best-seller in 2002*].

In these sentences, the presuppositions that are triggered by the italicized expressions are all inhibited. Thus, sentence (14) does not presuppose that Sue is working at home now; (15) does not presuppose that Sue has any married children (although it asserts that she has children); (16) does not presuppose that Rowling had a best-seller in 2002 (which in fact she didn't). The information that leads to these inhibitions is conveyed as part of the sentence. However, unlike sentences (9a-c), where the inhibitor of the second operand's presupposition is the first operand, the inhibiting information in (13)-(16) comes from different preceding parts within the sentence. Schematically, these sentences are all of the form X op_1 [Y op_2 Z], where the information that inhibits the presupposition in Z combines bits of information from X and Y.

How does this combination take place, and how is it related to incrementality and determinant values? To answer these questions, it is useful to employ another influential idea by Karttunen. In K74 he proposed to combine logical forms from different parts of a sentence into a *local context* (LC). The LC is responsible for inhibiting ("satisfying") or projecting any presupposition in its scope. For example, in (14)-(16) the relevant LCs for the italicized presupposition triggers are the following statements, respectively:

(17) Sue is at home; she is working now.

(18) Sue is miserably married with miserable kids; Sue's children get married.

(19) All of Rowling's Harry Potter books have been best-sellers; Rowling published a Harry Potter book in 2002.

These LCs are said to *satisfy* the corresponding presuppositions in (14)-(16) due to the following entailments, respectively:

(20) Sue is at home
Sue is working
⇒ Sue is at home working

(21) Sue has miserable kids
Sue's children get married
⇒ Sue has miserable kids who are in marriage relations

(22) All of Rowling's Harry Potter books have been best-sellers
Rowling published a Harry Potter book in 2002
⇒ Rowling had a best-seller in 2002

The operators in sentences (14) and (15) are conjunction and implication, and we see that their respective LCs (17) and (18) are formed by conjunction, in accordance with the observations above on (9a) and (9b). By contrast, the underlined assertion in (19) and (22) highlights the crucial effect of the disjunction in sentence (16). This sentence is of the form X *and* [Y *or* Z], and the relevant entailment in (22) involves *negating* the disjunctive negative operand Y (="Rowling published no Harry Potter book in 2002"). This is what we expect from Inferential Incrementality with disjunctions. Thus, aiming at a full treatment of propositional constructions and the explanatory problems that they raise, we focus on the following claim:

Claim: *A principled solution of the proviso problem emerges as a simple adaptation of Inferential Incrementality to LCs.*

This proposal differs from previous works on LCs that model them as sets of possible worlds, and treat satisfaction as set inclusion (Stalnaker, 1978; Peters, 1979; Rothschild, 2011; Nouwen et al., 2016). As Peters noted, this denotational approach leads to the same C-analysis as trivalent accounts. Thus, possible world accounts are also challenged by the proviso problem. The opposition between the inferential approach and denotational accounts remains when LCs enter the picture. One work that follows K74's inferential account as a solution to the proviso problem is (Mandelkern, 2016a). As in K74, Mandelkern does not fully define the logical forms in his system. The present paper agrees with Karttunen and Mandelkern's works in its representational approach to LCs and the inferential approach to satisfaction, but it is more explicit on the implications of this approach for a propositional fragment and its explanatory value. This propositional fragment, Karttunen Logic, is defined in section 4.

3.3. A note on a historical misconception

Throughout the discussion I have contrasted K73 and K74 with denotational accounts of presupposition. Against the background of previous work, this description may seem confusing. Following Peters (1979), many works have assumed that K74's "way of calculating presuppositions for the truth conditional connectives is equivalent to what would be obtained within a three-valued logic" (Beaver and Geurts, 2014: p.25). This is a misconception that seems to originate in Peters's claim that "[Karttunen (1974)] fallaciously argued that the regularities he discovered [...] cannot be embodied in any three-valued logic". However, the most apparent fallacy is in the way Peters ignores the empirical aspects of Karttunen's discovery, namely: the proviso problem. This was Karttunen's main argument for his inferential analysis and against trivalent denotational accounts (see note 2 above and K74, p.181). Peters showed that K74's rules can be *interpreted* using sets of possible worlds in a way that mimics trivalent truth tables. However, as we have seen, this interpretation does not conform with the predictions of K73 and K74's inferential mechanisms.

On a related note, Heim (1983: p.252) and Beaver (2001: p.82) mention K74 in relation to their assumption that presuppositions can be derived from admittance conditions. Both Heim and Beaver stipulate that a sentence S presupposes P if and only if all contexts that admit S entail P. By contrast, K74 defined the relation of presupposition-satisfaction without a formal notion of presupposition, warning against "confusing presuppositions with features of context that satisfy [them]" (p.192). Concerning his example *John called Mary a republican and then she insulted him back*, Karttunen writes: "there is nothing in [this example] which presupposes that 'Republican' is a dirty word". This is in contrast with the C-analyses by Heim and Beaver, which derive for this sentence the conditional presupposition *if John called Mary a republican then he insulted her*. In K74, a context that supports this conditional is "most obvious", but in contrast to his work from 1973, Karttunen (1974) does not proceed to defining presuppositions. For a similar point on K74 and its representation in subsequent literature, see (Francez, 2019).

4. Karttunen logic

In the previous section we discussed the accumulation of local contexts in sentences (14)-(16). It was noted that the distinction that these sentences show between disjunction on the one hand, and conditionals and conjunction on the other hand, is similar to what we might expect from incrementality-based analyses. Indeed, Denotational Incrementality (10) directly accounts for this similarity. To see that, let us reconsider sentence (14):

(23) *If* Sue is at home, *then* [she is working now and *Dan knows that Sue is at home working*].

If Sue is not at home, Denotational Incrementality expects (23) to be trivially true. If Sue is at home and not working, (23) is expected to be trivially false. Otherwise, the presupposition "Sue is at home working" of the last conjunct is true. Thus, presupposition inhibition is correctly predicted. K74 uses local contexts to achieve the same effect. His mechanism derives the conjunction *Sue is at home and Sue is working* as the LC of the presupposition trigger ("Dan knows that Sue is at home working"), and the entailment in (20) makes sure that the presupposition is satisfied in this LC. How does LC derivation come about, and what is its relation to the principle of Inferential Incrementality (12)? Answering these questions involves spelling out the definitions of LCs and satisfaction, hence the proposed *Karttunen Logic* (K-logic).

4.1. Preview

The key to the definition of K-logic is the concept of determinant value that underlies both incrementality principles above. In a binary operation, a determinant value of an operand is a value that fixes the result, no matter what the other operand says. Formally:

(24) When α is the lefthand operand of a binary operator op, we say that α **left-determines** the result of the operation if the following biimplication holds:

$$(\alpha\,\mathsf{op}\,\bot) \leftrightarrow (\alpha\,\mathsf{op}\,\top)$$

The standard notations '$\top$' and '$\bot$' are for tautological/contradictory propositions. The biimplication (24) requires that α *has a left-determinant value with respect to* op, which we abbreviate '$\mathrm{LDV}_{op}(\alpha)$'. By standard propositional identities, we get:

(25) $\mathrm{LDV}_{\rightarrow}(\alpha) = \mathrm{LDV}_{\wedge}(\alpha) = \neg\alpha$ *and* $\mathrm{LDV}_{\vee}(\alpha) = \alpha$

In words: a proposition α has an implicative/conjunctive LDV if α's negation is true; for α to have a disjunctive LDV, α itself has to be true.

Any sentence is analyzed with a *bivalent proposition C* modeling its local context. For a binary construction in a context *C*, the second operand is evaluated in a context that is updated using the negation of the first operand's LDV. This is defined below:

Update: Given an operation $\alpha\,\mathsf{op}\,\beta$, the *update of C by α with respect to* op is the proposition $C \wedge \neg\mathrm{LDV}_{op}(\alpha)$.

Context updating is the key to calculating the presuppositions of a complex formula, which is obtained by simplifying its elementary formulas α_{p} in their different LCs. The LC of each such formula then satisfies or dissatisfies the presupposition p according to the following definition:

Satisfaction: For any context C and elementary formula α_{p}, there are two possibilities:

(a) C entails p: in this case we say that C *satisfies* p. As a result, α_{p} is interpreted in context C as α – the assertive content of α_{p} without the presupposition.

(b) C does not entail p: C *dissatisfies* p, and then α_{p} is interpreted in context C with the presupposition p intact.

The intuition behind the Update and Satisfaction rules can be appreciated by considering the following facts on any binary construction $\alpha \,\mathsf{op}\, \beta_{\mathsf{p}}$:

- If the bivalent proposition α left-determines the result of the operation op, the expression $\text{LDV}_{op}(\alpha)$ is true. Thus, updating the context C by $\neg\text{LDV}_{op}(\alpha)$ leads to a *false* context.
- If α does not left-determine the result of the operation, the original context C remains intact after its update by $\neg\text{LDV}_{op}(\alpha)$.

Thus, to see if a context C satisfies the presupposition p in a binary construction $\alpha \,\mathsf{op}\, \beta_{\mathsf{p}}$, we only need to look at cases where α does not left-determine the result of the operation. This generalizes Inferential Incrementality (12) for LCs. Let us illustrate that by analyzing sentence (23) using Update and Satisfaction. We denote:

H = *Sue is at home*
W = *Sue is working*
$K_{H\wedge W}$ = *Dan knows that Sue is at home working*

Thus, sentence (23) is represented:

(26) $H \rightarrow (W \wedge K_{H\wedge W})$

To analyze (26), we start with an empty (tautological) context $\top$. This context is updated by the first operand H in (26), with respect to the implication operator. Thus, the context for the interpretation of the consequent in (26) is:

$$\top \wedge \neg\text{LDV}_{\rightarrow}(H) \;=\; H$$

This context, H, is updated by the operand W in (26) with respect to the conjunction operator:

$$H \wedge \neg\text{LDV}_{\wedge}(W) \;=\; H \wedge W$$

The updated context $H \wedge W$ satisfies the presupposition of $K_{H\wedge W}$, as intuitively required.

The reliance on determinant values when updating the context makes sure that disjunctions as in sentence (16) are treated on a par with the conjunction and implication in (23). To illustrate that, we represent sentence (16) using the following notations:

HP^{BS} = *all the Harry Potters were best-sellers*
$\neg HP^{02}$ = *no Harry Potter was published in 2002*

In this case, the chain of context updates for the conjunction and disjunction in (16) is:

$$\top \wedge \neg\text{LDV}_{\wedge}(HP^{\text{BS}}) \wedge \neg\text{LDV}_{\vee}(\neg HP^{02}) \quad = HP^{\text{BS}} \wedge HP^{02}$$

= *all the Harry Potters were best-sellers, and* <u>*some*</u> *Harry Potter was published in 2002*

In accordance with intuition, this context inhibits what the second disjunct in (16) counterfactually presupposes: that some Rowling book was a best-seller in 2002.

4.2. Formal definitions

K-logic is a trivalent propositional calculus that formalizes the projection algorithm sketched above. The elementary formulas in K-logic are pairs of a presupposition and an assertion, each of which is specified in the standard (bivalent) propositional calculus. Complex formulas are combinations of these elementary formulas using the propositional connectives. These elementary and complex formulas are referred to as *K-formulas*. The K-logic calculus computes the *assertion* $\boldsymbol{A}(\kappa)$ of any K-formula κ, as well as its *presupposition* $\boldsymbol{P}(C[\kappa])$ relative to a bivalent propositional context C.

Formally, the set of **K-formulas** is the smallest set containing the following formulas:

$(\mathsf{p}:\alpha)$	for any bivalent propositional formulas p and α
$\neg\varphi$	for any K-formula φ defined inductively
$\varphi \,\mathsf{op}\, \psi$	for any K-formulas φ and ψ defined inductively, where op is a binary propositional operator

The notation '$(\mathsf{p}:\alpha)$' for elementary K-formulas is a convenient synonym to 'α_{p}'. When a proposition α is asserted without a presupposition, we use the formula $(\top:\alpha)$. For example, more accurately than in (26), sentence (23) is represented using the following K-formula:

(27) $(\top:H) \rightarrow ((\top:W) \wedge (H \wedge W : B^{\text{HW}}))$

The two '$\top$'s in (27) represent the assumption that the sentences *Sue is at home* and *Sue is working now* are presupposition-less. *Dan knows that Sue is at home working* is represented $(H \wedge W : B^{\text{HW}})$, with the factive presupposition $H \wedge W$ and the assertive content B^{HW}= "Dan believes $H \wedge W$".[8]

For any K-formula κ, κ's **assertion** is a bivalent proposition $\boldsymbol{A}(\kappa)$, inductively defined by:

$$\boldsymbol{A}((\mathsf{p}:\alpha)) = \alpha$$

$$\boldsymbol{A}(\neg\varphi) = \neg\boldsymbol{A}(\varphi)$$

$$\boldsymbol{A}(\varphi \,\mathsf{op}\, \psi) = \boldsymbol{A}(\varphi) \,\mathsf{op}\, \boldsymbol{A}(\psi)$$

This definition simply combines the elementary assertions using the corresponding bivalent operators. For example, the K-formula (27) asserts the bivalent proposition $H \rightarrow (W \wedge B^{\text{HW}})$. This definition of the $\boldsymbol{A}$ operator reflects the fact that a sentence's assertive content is not affected by what its parts presuppose. For instance, let us consider the following sentences:

(28) If Sue stops smoking, she will join Dan in his aerobic training.

(29) If Sue doesn't smoke, she will start joining Dan in his aerobic training.

These sentences minimally differ in their presuppositions: (28) presupposes that Sue used to smoke, whereas (29) presupposes that so far she has not joined Dan's training. However, both sentences make the same assertion, namely: *if Sue doesn't smoke, she will join Dan's training*.

Although presuppositions do not affect the assertive content, we have seen that assertive contents may definitely affect presuppositions. One example is the contrast in presuppositions between sentences (9a) and (11a), which is caused by the change in the antecedent's assertion ("Sue is married" vs. "Sue is happy"). Incremental approaches capture these effects using the

[8]Knowledge is more than a belief that conforms with the facts, but this does not matter for the analysis.

Update and Satisfaction procedures. In K-logic, these procedures are the core of the definition of presuppositions relative to context. Formally, for any K-formula κ and bivalent context C, we define κ's **presupposition relative to** C as the bivalent proposition $\boldsymbol{P}(C[\kappa])$, inductively defined by the following rules:

(R1) $\boldsymbol{P}(C[(\mathsf{p}:\alpha)]) = \begin{cases} \top & C \Rightarrow \mathsf{p} \\ \mathsf{p} & \text{otherwise} \end{cases}$

(R2) $\boldsymbol{P}(C[\neg\varphi]) = \boldsymbol{P}(C[\varphi])$

(R3) $\boldsymbol{P}(C[\varphi \,\mathsf{op}\, \psi]) = \boldsymbol{P}(C[\varphi]) \wedge \boldsymbol{P}(C'[\psi])$, where $C' = C \wedge \boldsymbol{P}(C[\varphi]) \wedge \neg\text{LDV}_{op}(\boldsymbol{A}(\varphi))$

In words:

- Rule (R1) defines the presupposition of an elementary K-formula $\kappa = (\mathsf{p}:\alpha)$ to be *satisfied* by the context C if and only if C entails p. In this case, κ's presupposition in C is tautological, i.e. semantically null. Otherwise, κ's presupposition in C is left intact as p.
- Rule (R2) standardly defines the presupposition of a negative K-formula $\neg\varphi$ in a context C to be the same as φ's presupposition in C.
- Rule (R3) defines the presupposition of a binary K-formula $\varphi \,\mathsf{op}\, \psi$ in a context C by conjoining φ's presupposition in C with ψ's presupposition in an updated context C'. This updated context C' conjoins C with φ's presupposition in C and the negation of the LDV operator applied to φ's assertion. This makes sure that C' is *false* in situations where φ's assertion's value left-determines the result of the operation. Consequently, these situations are ignored when C' is used for testing satisfaction of the elementary formulas in ψ.

4.3. Examples

The formal analyses in examples 1 and 2 below illustrate two key elements of K-logic: its ability to inhibit (or "filter") embedded presuppositions without generating the proviso problem, and its reliance on accumulative LCs.

Example 1: ***filtering without proviso***

Let us reconsider sentences (9a) and (11a), which are restated below:

(30) If Sue is *married/happy*, she brought *her spouse* to the party.

Abbreviations:[9]

MH = Sue is married/happy *S = Sue has a spouse* *BS = Sue brought a spouse of hers*

In a null context ($\top$), the presupposition of the sentences in (30) is calculated as follows:

$\boldsymbol{P}(\top[(\top:MH) \rightarrow (S:BS)])$

$= \boldsymbol{P}(\top[(\top:MH)]) \wedge \boldsymbol{P}((\top \wedge \boldsymbol{P}(\top[(\top:MH)]) \wedge \neg\text{LDV}_{\rightarrow}(\boldsymbol{A}((\top:MH))))[(S:BS)])$ *(by R3)*

$= \top \wedge \boldsymbol{P}((\top \wedge \top \wedge \neg\text{LDV}_{\rightarrow}(\boldsymbol{A}((\top:MH))))[(S:BS)])$ *(by R1* $\mathbf{P}(\top[(\top:MH)]) = \top$*, since* $\top \Rightarrow \top$*)*

$= \boldsymbol{P}((\neg\text{LDV}_{\rightarrow}(\boldsymbol{A}((\top:MH))))[(S:BS)])$

$= \boldsymbol{P}((\neg\text{LDV}_{\rightarrow}(MH))[(S:BS)])$ *(since* $\mathbf{A}((\top:MH)) = MH$*)*

[9]It is assumed here that definites presuppose the non-emptiness of their description and introduce an existential quantifier as their assertive content. Uniqueness effects are ignored, but the analysis does not hinge on that.

$= \boldsymbol{P}(MH[(S\!:\!BS)])$ *(by (24),* $\text{LDV}_{\rightarrow}(MH) = \neg MH$*)*

In the first case in (30), being married entails having a spouse, hence $MH \Rightarrow S$, and by (R1):

$\boldsymbol{P}(MH[(S\!:\!BS)]) \;=\; \top$

Thus, out of context (9a) is correctly expected to have no presupposition, as a case of "filtering". In the second case in (30), being happy doesn't entail having a spouse, hence by (R1):

$\boldsymbol{P}(MH[(S\!:\!BS)]) \;=\; S$

Thus, out of context, sentence (11a) is expected to have the presupposition *Sue has a spouse*, which correctly describes it as a case of "projection", avoiding the proviso problem.

Example 2: ***accumulative LCs***

Let us reconsider sentence (16), restated below:

(31) All of Rowling's Harry Potter books have been best-sellers, *and* [either Rowling published no Harry Potter book in 2002 or *it was* Harry Potter and the Goblet of Fire *that was a Rowling's best-seller in 2002*].

Abbreviations:[10]

HP^{BS} = *all Harry Potters were best-sellers* $\neg HP^{02}$ = *no H.P. was published in 2002*

BS^{02} = *Rowling had a best-seller in 2002* GB^{02} = *Goblet of Fire was best-seller in 2002*

In a null context, the presupposition of (31) is calculated as follows:

$\boldsymbol{P}(\top[(\top\!:\!HP^{\text{BS}}) \wedge ((\top\!:\!\neg HP^{02}) \vee (BS^{02}\!:\!GB^{02}))])$

$= \boldsymbol{P}(\top[(\top\!:\!HP^{\text{BS}})]) \wedge$
$\quad \boldsymbol{P}((\top \wedge \boldsymbol{P}(\top[(\top\!:\!HP^{\text{BS}})]) \wedge \neg\text{LDV}_{\wedge}(\boldsymbol{A}((\top\!:\!HP^{\text{BS}}))))[(\top\!:\!\neg HP^{02}) \vee (BS^{02}\!:\!GB^{02})])$ *(by R3)*

$= \top \;\wedge\; \boldsymbol{P}((\top \wedge \top \wedge \neg\text{LDV}_{\wedge}(\boldsymbol{A}((\top\!:\!HP^{\text{BS}}))))[(\top\!:\!\neg HP^{02}) \vee (BS^{02}\!:\!GB^{02})])$
(since $\mathbf{P}(\top[(\top\!:\!HP^{\text{BS}})]) = \top$*)*

$= \boldsymbol{P}((\neg\text{LDV}_{\wedge}(\boldsymbol{A}((\top\!:\!HP^{\text{BS}}))))[(\top\!:\!\neg HP^{02}) \vee (BS^{02}\!:\!GB^{02})])$

$= \boldsymbol{P}((\neg\text{LDV}_{\wedge}(HP^{\text{BS}}))[(\top\!:\!\neg HP^{02}) \vee (BS^{02}\!:\!GB^{02})])$

$= \boldsymbol{P}(HP^{\text{BS}}[(\top\!:\!\neg HP^{02}) \vee (BS^{02}\!:\!GB^{02})])$ *(by (24),* $\text{LDV}_{\rightarrow}(HP^{\text{BS}}) = \neg HP^{\text{BS}}$*)*

$= \boldsymbol{P}(HP^{\text{BS}}[(\top\!:\!\neg HP^{02})])$
$\quad \wedge \boldsymbol{P}((HP^{\text{BS}} \wedge \boldsymbol{P}(HP^{\text{BS}}[(\top\!:\!\neg HP^{02})]) \wedge \neg\text{LDV}_{\vee}(\boldsymbol{A}((\top\!:\!\neg HP^{02}))))[(BS^{02}\!:\!GB^{02})])$ *(by R3)*

$= \top \wedge \boldsymbol{P}((HP^{\text{BS}} \wedge \top \wedge \neg\text{LDV}_{\vee}(\boldsymbol{A}((\top\!:\!\neg HP^{02}))))[(BS^{02}\!:\!GB^{02})])$
(since $HP^{\text{BS}} \Rightarrow \top$*, by R1:* $\mathbf{P}(HP^{\text{BS}}[(\top\!:\!\neg HP^{02})]) = \top$*)*

$= \boldsymbol{P}((HP^{\text{BS}} \wedge \neg\text{LDV}_{\vee}(\boldsymbol{A}((\top\!:\!\neg HP^{02}))))[(BS^{02}\!:\!GB^{02})])$

$= \boldsymbol{P}((HP^{\text{BS}} \wedge \neg\text{LDV}_{\vee}(\neg HP^{02}))[(BS^{02}\!:\!GB^{02})])$

$= \boldsymbol{P}((HP^{\text{BS}} \wedge HP^{02})[(BS^{02}\!:\!GB^{02})])$ *(by (24),* $\text{LDV}_{\vee}(\neg HP^{02}) = \neg HP^{02}$*)*

$= \top$ *(*$HP^{\text{BS}} \wedge HP^{02} \Rightarrow BS^{02}$*, since "all H.P.'s were best-sellers and some H.P. was published in 2002" entails "Rowling had a best-seller in 2002")*

Thus, in a null context sentence (31) is correctly expected to not to have any presuppositions.

Another aspect of LCs in K-logic is that in addition to assertive contents, they also accumulate the presuppositions into the LC (Karttunen, 1973: p.177). This is expressed in the second

[10] A cleft *it was NP that VP* is standardly assumed to presuppose *there was something that VP* and to assert *NP VP*.

clause of rule (R3) that updates the context C in binary constructions:

$$C' = C \wedge \underline{\boldsymbol{P}(C[\varphi])} \wedge \neg \mathrm{LDV}_{op}(\boldsymbol{A}(\varphi))$$

Example 3 below illustrates that by analyzing the following sentence:

(32) If Sue stopped smoking then Dan knows that Sue stopped smoking.

In sentence (32), the antecedent presupposes that Sue used to smoke and asserts that Sue does not smoke now. Due to the factive *know*, the consequent in (32) presupposes the conjunction of these two propositions. Sentence (32) as a whole only projects the presupposition of its antecedent ("Sue used to smoke"). According to the proposed K-logic, this happens because the presupposition of the consequent in (32) is satisfied by its LC, which includes both the assertive content and the presupposition of the antecedent.

Example 3: ***presuppositions are part of the LC***

Abbreviations:

US = *Sue used to smoke* S = *Sue smokes now*

B^{USns} = *Dan believes that Sue used to smoke and doesn't smoke now*

In K-logic, the presupposition of (32) in a null context $\top$ is calculated as follows:

$\boldsymbol{P}(\top[(US{:}\neg S) \to (US \wedge \neg S : B^{\mathrm{USns}})])$

$= \boldsymbol{P}(\top[(US{:}\neg S)]) \wedge \boldsymbol{P}((\top \wedge \boldsymbol{P}(\top[(US{:}\neg S)]) \wedge \neg\mathrm{LDV}_{\to}(\boldsymbol{A}((US{:}\neg S))))[(US \wedge \neg S : B^{\mathrm{USns}})])$ *(by R3)*

$= US \wedge \boldsymbol{P}((US \wedge \neg\mathrm{LDV}_{\to}(\boldsymbol{A}((US{:}\neg S))))[(US \wedge \neg S : B^{\mathrm{USns}})])$ *(since* $\mathbf{P}(\top[(US{:}\neg S)]) = US$*)*

$= US \wedge \boldsymbol{P}((US \wedge \neg\mathrm{LDV}_{\to}(\neg S))[(US \wedge \neg S : B^{\mathrm{USns}})])$

$= US \wedge \boldsymbol{P}((US \wedge \neg S)[(US \wedge \neg S : B^{\mathrm{USns}})])$ *(by (24),* $\mathrm{LDV}_{\to}(\neg S) = S$*)*

$= US \wedge \top$ *(by R1)*

$= US$

The initial motivation for developing K-logic is the distinction between presuppositions and admittance conditions (section 2.1). To see how this distinction is captured, let us reconsider sentence (2) and (13), which are restated below:

(33) If Sue visits Dan, she will like his beard.

(34) If Sue visits Dan he will have a beard, *and* [*if* she visits him, she will like his beard].

As we have seen, sentence (33) presupposes that Dan has a beard. This presupposition is inhibited in sentence (34). Thus, the first conjunct of (34) is characterized as an admittance condition of (33). In K-logic, this is captured because in a null context the derived presupposition of (34) is tautological, as formally shown in example 4 below.

Example 4: ***admittance conditions***

Abbreviations:

V = *Sue visits Dan* B = *Dan has a beard* L = *Sue likes a beard that Dan has*

In K-logic, the presupposition of (34) in a null context is calculated as follows:

$$\begin{aligned}
&\boldsymbol{P}(\top[((\top{:}V)\to(\top{:}B))\wedge((\top{:}V)\to(B{:}L))])\\
&=\boldsymbol{P}(\top[(\top{:}V)\to(\top{:}B)])\\
&\quad\wedge\boldsymbol{P}((\top\wedge\boldsymbol{P}(\top[(\top{:}V)\to(\top{:}B)])\wedge\neg\text{LDV}_{\wedge}(\boldsymbol{A}((\top{:}V)\to(\top{:}B))))[(\top{:}V)\to(B{:}L)]) \quad \textit{(by R3)}\\
&=\top\wedge\boldsymbol{P}((\top\wedge\top\wedge\neg\text{LDV}_{\wedge}(\boldsymbol{A}((\top{:}V)\to(\top{:}B))))[(\top{:}V)\to(B{:}L)])\\
&\qquad\qquad \textit{(since } \mathbf{P}(\top[(\top{:}V)\to(\top{:}B)])=\top\textit{)}\\
&=\boldsymbol{P}((\neg\text{LDV}_{\wedge}(\boldsymbol{A}((\top{:}V)\to(\top{:}B))))[(\top{:}V)\to(B{:}L)])\\
&=\boldsymbol{P}((\neg\text{LDV}_{\wedge}(V\to B))[(\top{:}V)\to(B{:}L)])\\
&=\boldsymbol{P}((V\to B)[(\top{:}V)\to(B{:}L)]) \quad \textit{(by (24), } \text{LDV}_{\to}(V\to B)=\neg(V\to B)\textit{)}\\
&=\boldsymbol{P}((V\to B)[(\top{:}V)])\wedge\boldsymbol{P}(((V\to B)\wedge\boldsymbol{P}((V\to B)[(\top{:}V)])\wedge\neg\text{LDV}_{\to}(\boldsymbol{A}((\top{:}V))))[(B{:}L)]) \quad \textit{(by R3)}\\
&=\top\wedge\boldsymbol{P}(((V\to B)\wedge\top\wedge\neg\text{LDV}_{\to}(\boldsymbol{A}((\top{:}V))))[(B{:}L)]) \quad \textit{(since } \mathbf{P}((V\to B)[(\top{:}V)])=\top\textit{)}\\
&=\boldsymbol{P}(((V\to B)\wedge\neg\text{LDV}_{\to}(\boldsymbol{A}((\top{:}V))))[(B{:}L)])\\
&=\boldsymbol{P}(((V\to B)\wedge\neg\text{LDV}_{\to}(V))[(B{:}L)])\\
&=\boldsymbol{P}(((V\to B)\wedge V)[(B{:}L)]) \quad \textit{(by (24), } \text{LDV}_{\to}(V)=\neg V\textit{)}\\
&=\top \quad \textit{(by R1, since } (V\to B)\wedge V\Rightarrow B\textit{)}
\end{aligned}$$

This correctly characterizes the conditional *if Sue visits Dan he will have a beard* as an admittance condition of sentence (33).

As we have seen, denotational approaches lead to a C-analysis that derives sound admittance conditions, but have to strengthen them in order to derive empirically sound presuppositions. In K-logic, presuppositions are derived directly. Admittance conditions are indirectly modelled as in example 4. A natural question is whether there is any difference between the admittance conditions that are expected in this way and those that are derived by C-analyses. With respect to the most popular C-analyses, either trivalent (Peters, 1979) or satisfaction-based (Nouwen et al., 2016), the answer is negative – for any K-formula, the weakest admittance conditions in K-logic are the same as those that are derived by C-analyses (Winter, 2020). If we accept the C-analysis as a proper theory of admittance conditions, this fact means that K-logic makes felicitous predictions about them, but without problematically conflating them with presuppositions.

5. Conclusions and open questions

The inferential approach to presupposition satisfaction relies on similar principles to those that underly Denotational Incrementality. By relying on global entailment relations between propositions, it disentangles presupposition from admittance, thereby avoiding the proviso problem for denotational approaches. These advantages come with a complicating factor: the reliance on inferences requires deriving sentence representations as part of presupposition projection. Using the proposed Karttunen Logic, this paper has shown how such derivations might work in the propositional domain. In this system, local contexts are treated as propositional formulas that are incrementally constructed as part of sentence processing.

To examine the generality of this inferential approach, we should of course like to extend this propositional fragment to non-sentential phenomena while preserving its explanatory advantages. There are three challenges I would like to mention in this respect. First, the domain of presupposition inhibition does not have to be sentential. This is illustrated by the following sentence:

(35) If Sue is married then her aunt must be hating Sue's spouse.

Intuitively, sentence presupposes that Sue has an aunt, but not that she has a spouse. Under the inferential approach, this "selective" inhibition should be modeled at the sub-sentential trigger's site, i.e. the phrase *Sue's spouse*. This is because the presupposition of the consequent in (35) is *Sue has an aunt and a spouse*, which is not entailed by the antecedent (Sue might be married but aunt-less). Thus, satisfaction of the presupposition of *Sue's spouse* would not go through in (35) if it applied at the propositional level.

Secondly, as noted by Schlenker (2009), local contexts might also be non-propositional. For instance, in *no drug addict stopped using drugs voluntarily*, the predicative presupposition *used drugs* is satisfied by the non-propositional local context that is created by the quantifier. Schlenker concludes that local contexts should be generalized to all types. However, it is unclear that this is actually how things work in cases like the following:

(36) Global context: all of Sue's Swede boyfriends have been blonds.
If Sue's current boyfriend is Swede, he must have dyed <u>his blond hair</u> *black.*

In this case, the presupposition that Sue's boyfriend has blond hair is inhibited. This inhibition is a combination of a propositional global context and the predicative local context created by *Swede*. Separating local contexts of different types does not model this kind of combination.

Thirdly, there is the question of attitude reports. As Heim (1992) mentions, the distinction between admittance and presuppositions persists in sentences like the following:

(37) Dan wants Sue to stop smoking.

Out of the blue, the conclusion from (37) is that Sue smokes. However, this sentence is admitted in contexts where Dan only incorrectly believes that Sue smokes. This is similar to the proviso problem with propositional connectives. Consistently with her C-analysis, Heim treats cases like (37) as presupposing that Dan believes that Sue smokes, and explains the inference that follows from (37) out-of-the-blue using pragmatic considerations. In the inferential approach, however, we might expect the reasoning to be reversed: contexts where Dan believes that Sue smokes might be sufficient to inhibit a default presupposition of (37) about Sue's factual smoking habits. Whether this can be worked out in a principled way is left for further research.

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