On Inference with Scopally Ambiguous Sentences

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Plan of talk

1. Background on scope dominance and generalized quantifiers.
2. Scope dominance with upward-monotone quantifiers on countable domains.
3. Scope dominance with downward-monotone quantifiers (on finite domains).
4. Computing scope dominance.
5. Some open problems and afterthoughts.

Pointers

Talk based on the following downloadable papers:

1. Altman, Keenan and Winter:
   `http://www.cs.technion.ac.il/~winter/papers/ms.abs.html`
2. Altman, Peterzil and Winter:
   `http://www.cs.technion.ac.il/~winter/papers/msp.abs.html`
3. Altman and Winter:
   `http://www.cs.technion.ac.il/~winter/papers/csd.abs.html`
   With a demo system at:
   `http://lingua.cs.technion.ac.il/~alon_a`
4. Ben-Avi and Winter:
   `http:`
   `//www.cs.technion.ac.il/~winter/papers/sdmon.abs.html`

Entailments between narrow and wide scope readings

(1) every (some) student saw every (some) teacher
OWS $\Leftrightarrow$ ONS

(2) every student saw some teacher
OWS $\Rightarrow$ ONS

(3) some student saw every teacher
ONS $\Rightarrow$ OWS

(4) every/some student saw exactly one teacher
more than one student saw more than one teacher
OWS $\not\Rightarrow \not\Leftarrow$ ONS

Aim: Given a sentence S of the form NP1-TV-NP2, we would like to characterize whether OWS(S) entails (is entailed by) ONS(S).

Why are such entailments interesting?

- They pose nice challenges for generalized quantifier theory.
- They are part of theories about economy and scope (e.g. Fox, 1999).
- They are relevant for underspecification, inference under ambiguity and other cases of inference in computational semantics.

  Van Deemter, K. (1996). Towards a logic of ambiguous expressions. In Van Deemter, K. and Peters, S., editors, *Semantic Ambiguity and Underspecification*, CUP.
- Failures to identify them have led to considerable mess in the literature.

  Eddy Ruys, Wide Scope Indefinites: The Genealogy of a Mutant Meme.

  `http://www.let.uu.nl/~Eddy.Ruys/personal/linguist.htm`

  "How an innocent observation by Tanya Reinhart in (1976) initiated a string of publications that all managed to get a very simple observation wrong."

Generalized quantifiers and ONS/OWS readings

A *Generalized Quantifier* (GQ) over a non-empty domain E is a subset of $\wp(E)$.

$$Q_1{-}Q_2 \overset{def}{=} \{R \subseteq E^2 : \{x \in E : R_x \in Q_2\} \in Q_1\} \quad \text{(ONS)}$$
$$Q_1{\sim}Q_2 \overset{def}{=} \{R^{-1} : R \in Q_2{-}Q_1\} \quad \text{(OWS)}$$

A *global determiner* is a functor D s.t. for any non-empty domain E: $D_E \subseteq \wp(E) \times \wp(E)$.

Note: modulo isomorphism, D_E is a function from $\wp(E)$ to GQs over E.

$$D_1{-}D_2 \overset{def}{=} \{\langle A, B, R\rangle : R \in D_1(A){-}D_2(B)\}$$
$$D_1{\sim}D_2 \overset{def}{=} \{\langle A, B, R\rangle : R \in D_1(A){\sim}D_2(B)\}$$

Scope Dominance

For two generalized quantifiers Q_1 and Q_2, we say that Q_1 is *scopally dominant over* Q_2 if $Q_1{-}Q_2 \subseteq Q_1{\sim}Q_2$.

General question: Characterize the pairs of quantifiers in the scope dominance relation.

Alternatively: Characterize the pairs of (global) determiners D_1, D_2 such that $D_1{-}D_2 \subseteq D_1{\sim}D_2$.

Special cases of scope dominance

- Zimmerman (1993) fully characterizes the class of “scopeless” object GQs - those Q_1s for which Q_1–$Q_2 = Q_1{\sim}Q_2$ for every Q_2. This is the class of *principal ultrafilters* (names).
- Westerståhl (1996) fully characterizes the class of “self-commuting” GQs - those Qs for which Q–$Q = Q{\sim}Q$.

Duality and scope dominance

Complement of Q:

$$\overline{Q} \stackrel{def}{=} \wp(E) \setminus Q$$

Postcomplement of Q:

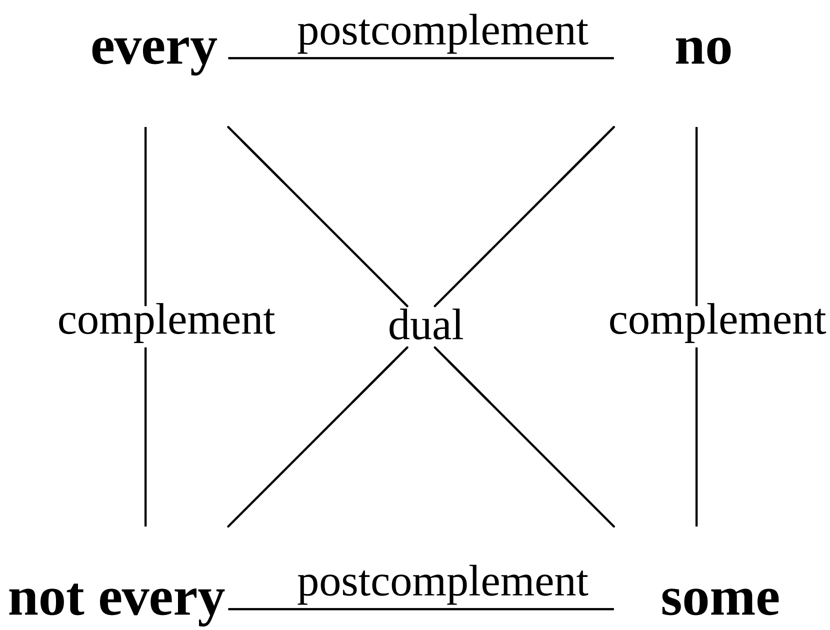
$$Q- \stackrel{def}{=} \{A \subseteq E : E \setminus A \in Q\}$$

Dual of Q:

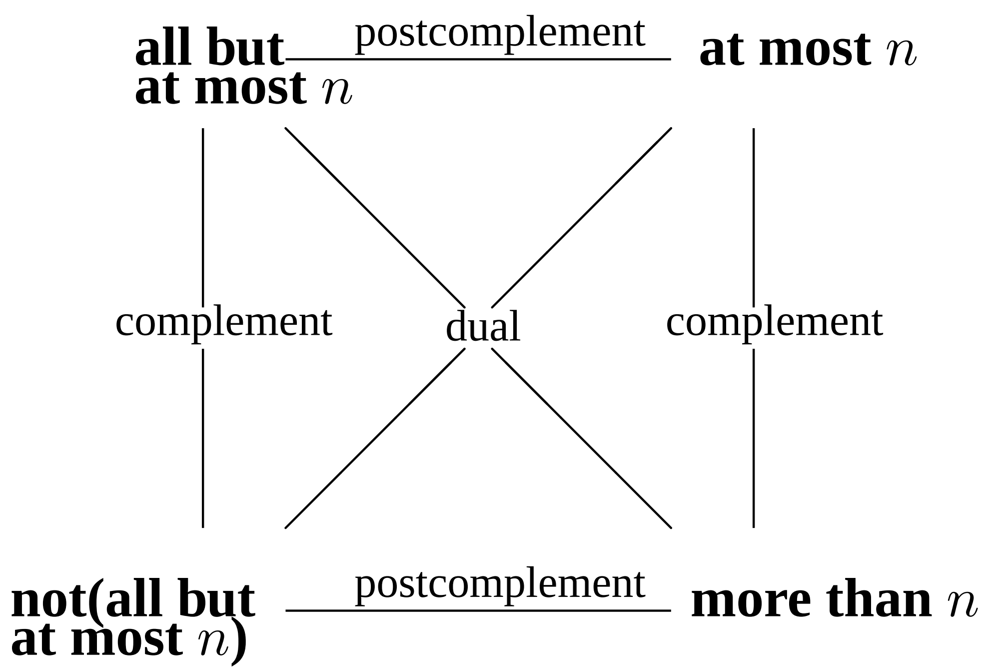
$$Q^d \stackrel{def}{=} \overline{(Q-)} = (\overline{Q})- = \{A \subseteq E : E \setminus A \notin Q\}$$

Fact: For all quantifiers Q_1 and Q_2: Q_1 is dominant over Q_2 iff Q_2^d is dominant over Q_1^d.

Example 1: the square of opposition



Example 2: the generalized square of opposition



When $n = 0$ this is the traditional square of opposition.

Example: duality and scope dominance

The entailment ONS$\Rightarrow$OWS in (2) follows by duality from the entailment ONS$\Rightarrow$OWS in (1):

(1) Some student saw more than three teachers.

(2) All but at most three teachers were seen by every student.

Westerståhl (1986): scope dominance of upward monotone quantifiers over finite domains

A quantifier Q_E is called *upward (downward) monotone* if for all $A \subseteq B$: $B \in Q$ if (only if) $A \in Q$.

$Q \stackrel{def}{=} UNIV(A)$ if $Q = \{B \subseteq E : A \subseteq B\}$;
$Q \stackrel{def}{=} EXIST(A)$ if $Q = \{B \subseteq E : A \cap B \neq \emptyset\}$.

We say that Q is *UNIV* (*EXIST*) if there is $A \subseteq E$ s.t. $Q = UNIV(A)$ ($Q = EXIST(A)$).

Note that $(EXIST(A))^d = UNIV(A)$.

Fact (Westerståhl 1986): Let Q_1 and Q_2 be **upward monotone** quantifiers over a **finite** domain E. Q_1 is dominant over Q_2 iff these quantifiers fall under at least one of the following cases.

(i) Q_1 is *EXIST* or Q_2 is *UNIV*.

(ii) $Q_1 = \wp(E)$ and $Q_2 \neq \emptyset$, or $Q_2 = \emptyset$ and $Q_1 \neq \wp(E)$.

Sketch of main part of proof

Assume that Q_1 is dominant over Q_2, and assume for contradiction that neither *(i)* nor *(ii)* holds. Then by finiteness of E there is a minimal set $A \in Q_1$ such that $|A| \geq 2$ (otherwise by upward monotonicity, $Q_1 = \wp(E)$ or $Q_1 = EXIST(\bigcup_{\{x\} \in Q_1} \{x\})$). By the dual consideration, there are $B_1, B_2 \in Q_2$ such that $B_1 \cap B_2 \notin Q_2$. Given the sets A, B_1 and B_2, and an arbitrary $a \in A$, it is easy to verify that the relation $(\{a\} \times B_1) \cup ((A \setminus \{a\}) \times B_2)$ contradicts our assumption that Q_1 is dominant over Q_2.

But in general, this characterization is too narrow for infinite domains

(5) Infinitely many cases are covered by item 1 or item 2.
Both item 1 and item 2 cover all but finitely many cases.

ONS$\Rightarrow$OWS, but Q_1 is not *EXIST* and Q_2 is not *UNIV*

Two directions:

1. Find principles that determine the special status of (*all but*) (*in*)*finitely many* in natural language – Altman, Keenan and Winter (2001).
2. Generalize Westerståhl's result – Altman, Peterzil and Winter (2002).

Properties of quantifiers over countable domains

A quantifier Q satisfies the *Descending Chain Condition* (DCC) if for every descending sequence $A_1 \supseteq A_2 \supseteq \cdots A_n \supseteq \cdots$ in Q, the intersection $\bigcap_i A_i$ is in Q as well.

Example: Any *UNIV* quantifier satisfies (DCC). A quantifier $EXIST(X)$ satisfies (DCC) if and only if X is finite.

Altman, Peterzil and Winter (2002)

Let Q_1 and Q_2 be upward monotone quantifiers over a countable domain E. Then Q_1 is dominant over Q_2 if and only if all of the following requirements hold:

(i) Q_1^d or Q_2 are closed under finite intersections;

(ii) Q_1^d or Q_2 satisfy (DCC);

(iii) Q_1^d or Q_2 are not empty.

Note: On finite domains (i) boils down to a *UNIV* requirement, and (ii) is trivially met – so this is Westerståhl’s (1986) characterization.

Examples

(6) Infinitely many cases are covered by item 1 or item 2.

Q_1^d is closed under finite intersections; Q_2 satisfies (DCC)

(7) Both item 1 and item 2 cover all but finitely many cases.

Q_1^d satisfies (DCC); Q_2 is closed under finite intersections

Downward monotone quantifiers over finite domains (Ben-Avi and Winter 2005)

Let Q_1 and Q_2 be two (non-trivial) quantifiers over a **finite** domain E, s.t. Q_1 is $\mathrm{MON}\!\downarrow$ and Q_2 is $\mathrm{MON}\!\uparrow$. Let

$n \stackrel{def}{=} \max\{|Y| : Y$ is minimal in $Q_1-\}$.

Then Q_1 is scopally dominant over Q_2 iff for every $Q \subseteq Q_2^d$, if $|Q| = n + 1$ then $\bigcap Q \neq \emptyset$.

and of course a dual result for $\mathrm{MON}\!\uparrow$-$\mathrm{MON}\!\downarrow$ pairs.

(1) <D in $\mathrm{MON}\!\downarrow$> *referees read at least one of the abstracts*

ONS⇒OWS: Q_2^d = *every abstract*, *the* guarantees non-triviality, hence $\bigcap Q_2^d \neq \emptyset$

(2) <D in $\mathrm{MON}\!\downarrow$> *referees read each of the abstracts*

OWS⇒ONS: by duality to (1)

(3) *More than half of the referees read no abstract*

ONS⇒OWS: by duality to (4)

(4) *Not every abstract was read by at least half of the referees*

ONS⇒OWS: Q_1- = *some abstract*, $n = 1$, Q_2^d = *more than half of the referees*

Computing scope dominance (Altman and Winter 2004)

Given a sentence of the form NP1-TV-NP2, how can we effectively *compute* whether the ONS reading entails the OWS reading?

Two problems:

1. Cardinality presuppositions may create scope dominance relations, even though in general the determiners alone do not support them.

   (1) At least two of the three persons in this room admire more than two students in this room.

   No general scope dominance between *at least two of the three* and *more than two*.
   But here, *more than two students in this room* is either empty or universal, hence ONS⇒OWS.

2. NP conjunctions where the live-on sets of the two conjuncts have a non-empty intersection can lead to more scope dominance relations than what is anticipated from the determiners alone.

(2) At least two priests met every teacher and at least two authors.

In general – no scope dominance.
But when there are at least two authors who are also teachers, (2) is equivalent with:

(3) At least two priests met every teacher.

An algorithm that partly deals with these problems is introduced in Altman and Winter (2004). For a demo see:

`http://lingua.cs.technion.ac.il/~alon_a`

Specifiying Determiner Functions

Fact 1 (Väänänen and D. Westerståhl, 2001): For any monotone conservative global determiner D that satisfies ISOM and EXT, there is a function $g_{det} : \mathbb{N} \rightarrow \mathbb{N}$ that satisfies for all $A, B \subseteq E$ s.t. A is finite:

$$B \in D(A) \Leftrightarrow |A \cap B| \geq g_{det}(|A|).$$

Fact 2: For countably infinite first arguments, we can use the property $inf_{det} \in \{$'at_least_n', 'infinitely_many', 'all_but_lt_n', 'all_but_fin_many'$\}$ to describe the behavior of any right-upward-monotone conservative determiner that satisfies ISOM and EXT.

Lowest General Cardinality (*lgc*)

Definition: Let D be a global ISOM determiner. The *lowest general cardinality* (*lgc*) of D is the minimal value in $\mathbb{N} \cup \{\aleph_0\}$ s.t. the class of generalized quantifiers $D_E(A)$ with $A \subseteq E$ and $|A| \geq lgc$ is contained in (exactly) one of the following four classes of GQs: TRIV, $\mathrm{EXIST} \setminus \mathrm{TRIV}$, $UNIV \setminus \mathrm{TRIV}$ or $\overline{\mathrm{EXIST} \cup UNIV}$.

For example:

$$\mathbf{more\_than\_half}(A) = \begin{cases} \emptyset & |A| = 0 \\ UNIV(A) & |A| = 1, 2 \end{cases}$$

And only when $|A| \geq 3$ is $\mathbf{more\_than\_half}(A)$ non-trivial and distinct from both *UNIV* and *EXIST*. Hence $lgc(\mathbf{more\_than\_half}) = 3$.

Values of g_{det}, inf_{det}, and *lgc* for some determiner expressions

Determiner	$g_{det}(n)$	inf_{det}	*lgc*
every, each, all	n	all_but_lt_1	2
some	1	at_least_1	2
at least half	$\lceil n/2 \rceil$	*undefined*	3
more than half	$\lceil (n+1)/2 \rceil$	*undefined*	3
all but at most m	$\max(n-m, 0)$	all_but_lt_$m+1$	$m+2$
infinitely many	$n+1$	infinitely_many	$\aleph_0$
all but finitely many	0	all_but_fin_many	$\aleph_0$

Essence of algorithm for computing scope dominance

Use the *lgc* and inf_{det} values of the determiners in transitive sentence, as well as given restrictions on cardinality of nominal denotations, to generate a *indicative model*: a model in which (lack) of scope dominance between the subject and object quantifiers is indicative for entailments between the ONS and OWS readings.

For further research

1. Full characterization of scope dominance:
   - With downward monotone and non-monotone unary quantifiers.
   - With other constructions (e.g. inverse linking).
   - Over (countable or non-countable) infinite domains.
2. Complete derivation of “scope entailments” within a fragment that contains coordination and cardinality presuppositions.
3. Implementation for reasoning under ambiguity.

Proof Theory at the Syntax/Semantics Interface

It’s the same paradigm of “Natural Logic” that underlies our approach to scope entailments: use the syntactic restrictions on natural language sentences, add to it knowledge about logical items in the language (quantifiers, coordinators), in order to describe its sentences’ inferential potentials.

A much more solid interface with Proof Theory is needed in order to achieve more general results along these lines.

But we may soon need to decide about our desiderata: would a clean proof theoretical formalism have to be at the expense of the “linguistic reality” of our theories?