Choice Functions and the Scopal Semantics of Indefinites

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1 Introduction

The exceptional scopal behaviour of indefinite NP's is a long-standing challenge for syntactic and semantic theories. Plural indefinites trigger also distributivity effects that introduce further complications in the problem of scope. Taking these interactions into account, the present paper proposes a revision in standard approaches to the syntax and semantics of indefinites. Following Reinhart's article "Quantifier Scope" (1996), we explain the unique scopal behaviour of simple plural and singular indefinites by a *choice function* semantics, and not by any scope shifting operation. Unlike Reinhart's approach, where also standard quantification is assumed, it is proposed that quantification over choice functions is the *uniform* mechanism for interpreting indefinites. Evidence is given from various phenomena that are not explained by standard scope mechanisms with generalized quantifiers. On the other hand, a syntactically restricted scope mechanism that was motivated for other NP's, is motivated for indefinites as well. Semantic operators for choice function and distributive interpretation apply to the output of this mechanism. The formal definition proposed for choice functions is shown to be linguistically adequate and workable within a compositional system. It turns out to extend (but not replace) the standard generalized quantifier treatment of indefinites.

The organization of the paper is as follows. Section 2 introduces some familiar theoretical background on the problem and previous proposals for solution. Section 3 informally presents the proposed theory of choice functions and indefinites and the linguistic arguments for it. Section 4 shows some of the difficulties for an adequate formal semantics of choice functions and proposes a solution to these problems. Section 5 suggests a general line for a compositional categorial semantic implementation of the proposed formalism. Section 6 briefly examines the relations between such a mechanism and generalized quantifier theory. Appendix A brings proofs of propositions. Appendix B contains a formal procedure for compositional interpretation along the lines of section 5.

Sections 2-3, section 4 and sections 5-6 constitute three distinct units, each based upon the former, but not vice versa. Readers with different interests can therefore safely concentrate on different units.

2 Background

2.1 On the notion of semantic scope

Most linguistic theories of semantic interpretation pay much attention to questions concerning the semantic scope of noun phrases in various constructions. Somewhat unfortunately, the term "scope", being a structural notion, is not very suitable for describing truth-conditional intuitions. Consider for example sentences (1) and (2). It is completely reasonable to describe the relation between the subject and the reflexive pronoun in (1) using a purely formal definition of the subject's scope (in terms of c-command relations, for example). But it is almost meaningless to say that sentence (2) has a reading in which the object takes scope over the subject, without making some assumptions about the *syntax* of an abstract representational level, quite remote from phonology.

(1) Every politician admires herself.

(2) Some citizen admires every politician.

Sentences like (2) are often analyzed as having a reading that can be expressed by the first-order logic proposition in (3).

(3) $\forall x[\mathbf{politician}'(x) \rightarrow \exists y[\mathbf{citizen}'(y) \wedge \mathbf{admire}'(y, x)]]$

In classical Montague semantics and in a large part of the literature on Logical Form following May (1977) it is assumed that some syntactic operation should intermediate between the surface structure of (2) and the proposition that (3) reflects. Roughly, this mechanism produces a syntactic representation as in (4).

(4) [every politician] [[some citizen] [admires]]

Interpretative mechanisms proposed in Montague semantics and Generalized Quantifier theory, as well as (more or less explicitly) in the Logical Form literature, derive the semantic proposition in (3) from a syntactic representation as in (4). Once such mechanisms are assumed, (3) can reasonably be called "a wide scope reading" of (2), because this proposition is derived by a mechanism on a syntactic level at which the object has standard (structural) wide scope over the subject.

If it were only for the interpretation of sentences like (2) in English, then the motivation for a syntactic representation like (4) would have been primarily semantic. In consideration of that, it has been often proposed that "wide scope" interpretations are a manifestation of semantic mechanisms in natural language which are significantly different from the linear quantification in classical predicate calculus as extended by Montague and the Generalized Quantifier tradition. Thus, semantic interpretation can apply to NP's in their surface order without further syntactic mechanisms, and still produce "wide scope" effects. This idea was implemented, among others by Cooper (1975) and, within a strictly compositional semantic system, by Hendriks (1993).

We can call the general mechanism (be it syntactic or semantic) that linguistic theory adopts for describing the scopal semantics of NP's by the name *the standard scope mechanism*. Whatever scope mechanism is used, one question of central linguistic importance has to do with the structural restrictions on such a mechanism. For example, a sentence like (5) does not have a reading in which the universal NP takes sentential scope: the sentence does not claim that the arrival of *any* woman will be sufficient for John to be glad. That is, (5a) is not a reading of (5).

(5) If every woman comes to the party John will be glad.

a. * $\forall x[\mathbf{woman}'(x) \rightarrow [\mathbf{come}'(x) \rightarrow \mathbf{glad}'(\mathbf{j}')]]$ (unavailable interpretation) [1]

Any theory of the scopal semantics of NP's should address then two different questions:

- What is the (syntactic/semantic) standard scope mechanism and what are the semantic properties of NP interpretation that allow its effect?
- What are the structural restrictions on the standard scope mechanism and how are they accounted for?

A growing consensus emerged in the literature since Lakoff (1970) and Rodman (1976) around the following empirical generalization: the restrictions on NP scope (at least) contain the structural constraints that govern extraction phenomena, known since Ross (1967) as "island constraints". This means that NP's, when viewed

[1] Here and henceforth, * indicates non-readings, $^{\#}$ indicates readings which are pragmatically "strange", and ***** and **#** indicate ungrammatical and pragmatically incoherent *sentences*.

as generalized quantifiers, do not show scope properties that would force them to cross island barriers once their scope is assigned syntactically as in (4). For example, the unavailability of reading (5a) for (5) is described by means of the same syntactic constraint that is used to describe the ill-formedness of (6), where a *wh* element is "extracted" from the adjunct island.

(6) ***** Who if comes to the party John will be glad?

This observation is most easily accounted for if the scope mechanism is a syntactic procedure, so that wide scope in (5) is blocked by the same structural principles that block (6). This is an important point in favour of a syntactic scope mechanism of Quantifier Raising or some other syntactic operation.

2.2 The scope problem of indefinites

An apparent exception to the observation about the similarity between structural restrictions on the scope mechanism and restrictions on other grammatical operations is the case of indefinite NP's. Since Fodor & Sag (1982), much attention has been paid to the fact that indefinites do not seem to be restricted by the constraints on the scope of other NP's. For example, (7) does have the reading (7a), [2] in contrast to (5).

(7) If some woman comes to the party John will be glad.

a. $\exists x[\mathbf{woman'}(x) \wedge [\mathbf{come'}(x) \rightarrow \mathbf{glad'}(\mathbf{j'})]]$

This "island escaping" scopal semantics of indefinites appears with all other islands for extraction. See Ruys (1992:102-103) for a concise description of the facts.

The scopal semantic behaviour of indefinites is a problem for any standard scope mechanism. It poses a question of fine-grainedness; namely, how to account for the seemingly exceptional behaviour of indefinites, while still preserving an account of the structural restrictions on the scope of other NP's? This problem is in the main focus of the present paper.

2.3 Previous accounts of the scope problem

A straightforward approach to the scope problem of indefinites is what I henceforth call the *free scope* approach. It has not been explicitly proposed by any work known to me. However, many authors (e.g. Fodor & Sag) use it as a possible idea, comparing it to their own proposals. "Free scope" simply means that indefinites are traditionally treated as existential *quantifiers* and that it is the standard scope mechanism that derives their wide scope effects, also beyond islands. Thus, it is stipulated that indefinites have some (possibly syntactic) feature that distinguishes them from other quantificational NP's.

Fodor & Sag (1982) argue against this approach. Instead, they propose that indefinites are *ambiguous*. On its standard quantificational reading, an indefinite NP behaves like any other quantificational NP. Especially, its scope is restricted by islands. However, indefinites also have a *referential* reading: the indefinite can function as "a 'private' pointing gesture within the mind of the speaker" (Fodor & Sag (1982:381)). In this reading the indefinite is supposed to behave semantically like a proper name. For proper names all scope construals are equivalent (see Zimmermann (1986,1991)). Fodor & Sag argue that this is indeed the case with the "island-escaping" effect of the scope of indefinites: it results in *only one* reading.

[2] Throughout this paper I treat conditionals using material implication, ignoring the well-known semantic/pragmatic problems concerning their correct interpretation. Of course, one may doubt whether (7a), which is verified by any situation in which there is one woman who did not come to the party, reflects correctly the wide scope reading of the indefinite in (7). Obviously, this problem is independent of the scope problem of indefinites. For this reason and because antecedents of conditionals are one of the simplest and most striking cases of scope islands, I use such examples freely, counting on the reader to substitute her favorite theory of conditionals for material implication.

This claim has been challenged in Farkas (1981), Rooth & Partee (1982:fn.6) and, more recently, in Ruys (1992) and Abusch (1994). These works all show cases where Fodor & Sag's claim is argued to be incorrect. The empirical debate will be reviewed later in this paper (subsection 3.4.2). Ruys and Abusch both conclude that Fodor & Sag's "referential" approach is inadequate. To handle the facts, Ruys proposes an indexing mechanism of indefinites within a DRT-like interpretation of LF. Abusch proposes to enrich DRT with a storage mechanism that changes the syntactic position of the N' predicate (= the *restriction* of the indefinite) at the representational level. Both Ruys and Abusch therefore accept the assumption of DRT about a distinct syntactic representational level for meaning. This level (sometimes called Logical Form') is additional to the syntactic level that undergoes semantic interpretation (GB's Logical Form, other theories' Surface Structure). Indefinites in Ruys and Abusch's treatments are not quantifiers. Instead, they involve the familiar treatment of DRT using free variables. I henceforth classify Ruys and Abusch's proposals as *representational* approaches to the scope problem. [3]

Another alternative is seminally proposed in Reinhart (1992) for *wh* elements and further developed in Reinhart (1996) also for non-interrogative indefinites. Reinhart proposes to use quantification over *choice functions* as the basic mechanism generating the wide scope of indefinites beyond islands. This idea shares with the representational approach the intuition that indefinites are not ordinary quantificational expressions. However, like Fodor & Sag, Reinhart proposes that also traditional existential quantifiers are among the readings of an indefinite. Reinhart's proposal will be discussed in detail in section 3.

Kratzer (1995) agrees with Reinhart about the need to distinguish a choice function analysis of indefinites from standard quantification. However, Kratzer does not accept the existential quantification over choice functions in Reinhart's proposal. For Kratzer, choice functions are *referential* entities, responsible for Fodor & Sag's "specific" reading of indefinites. Kratzer develops a counter-argument to the above mentioned arguments against Fodor & Sag (see subsection 3.4.2 below).

2.4 The plurality problem of numeral indefinites

A natural class of plural indefinites are NP's like *exactly six boys*, *less than five shops*, etc. I will henceforth concentrate on the subclass of (simple) *numeral indefinites* like *six boys* and *five shops*. Plural numeral indefinites introduce a problem that is a part of a classical problem for the semantics of plurals – the origins of distributivity and collectivity. Consider the following simple sentences:

(8) Three women smiled.

(9) Three women met.

In order to model the semantics of (8) we take into account the properties of *singular* individual women; ones like Hillary Clinton or Princess Diana. However, in (9) we have to consider properties of *collections* of women: individual women are normally not in the extension of *meet* (cf. the marked status of *?*Hillary Clinton met*). How is this difference reflected in the interpretation process of numeral indefinites? This problem will henceforth be referred to as *the plurality problem* of numeral indefinites.

2.5 Previous accounts of the plurality problem

The question of how to model "distributive" cases like (8) did not seem particularly troublesome to traditional logical approaches, as well as many versions of Generalized Quantifier theory. For example,

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[3] Other representational treatments of the scope problem appear in Farkas (1995) (using evaluation indices) and in Szabolcsi (1996) (using witness sets). To keep the discussion manageable I do not consider these alternatives, which are, however, similar in pre-assumptions and empirical predictions to the Ruys and Abusch proposals.

Barwise & Cooper (1981) propose to treat the NP in (8) as a quantifier over the domain of (singular) individuals. [4]

However, a greater challenge to contemporary semantics is the interpretation of (9) and its relations to (8). Most works on plurality [5] agree that (9) should reflect a proposition as in (10).

(10) $\exists x[\mathbf{three}'(x) \wedge \mathbf{women}'(x) \wedge \mathbf{meet}'(x)]$

In (10), the variable x ranges over "plural individuals". Roughly, this means that x can be a collection of individuals. For example, a possible value for x is the singular individual standing for *Mary*, but also a plural individual grouping together the individuals for *Mary*, *Sue* and *Jane*. The clause $\mathbf{three}'(x)$ means that x consists of exactly three singular individuals. The clause $\mathbf{women}'(x)$ means that each singular individual in x is a woman. The clause $\mathbf{meet}'(x)$ means that the plural individual x has the meeting property.

Much disagreement surrounds the question of how exactly to model plural individuals. [6] However, we are here interested in a different question: how do we derive the proposition in (10) from the syntactic representation of (9)? And how does this mechanism relate to the interpretation of (8)?

Bennett (1974) and Scha (1981) propose answers based on lexical ambiguity. The morpheme *three* in (8) and (9) corresponds to different lexical items. In both cases it is semantically a determiner: a function from predicates to generalized quantifiers. However, the two readings of *three* generate different quantifiers, designed to lead to the correct distributive/collective readings of the two sentences.

Van der Does (1992,1993) shares with Bennett and Scha the idea that *three* is a determiner. However, instead of lexical ambiguity, there are different operators that shift a basic lower type denotation of *three* into higher order determiners, similar to Scha's readings of *three*.

Another semantic approach to the problem takes numerals to be predicate *modifiers* (adjectives): functions from predicates to predicates. Thus, the construction *three women* basically denotes the set of collections that contain exactly three women. Many works (see below) adopt this approach to numerals, which has some familiar linguistic advantages over their treatment as determiners. [7] However, when *three women* denotes a predicate it cannot directly compose with the main predicate in the sentence as in the Bennett/Scha proposals.

One common resolution of this situation is to assume a phonologically null determiner position that lifts the predicate into an existential quantifier over plural individuals. This is the proposal in Link (1987). Verkuyl (1993) and Carpenter (1994a), among others, and it derives a "collective quantifier" as in Bennett and Scha's proposals. There are now some possibilities to derive distributivity in (8), but let us ignore this point. What is crucial for us is that also for Link, Verkuyl and Carpenter the NP ends up denoting a quantifier. Thus, I henceforth group this treatment together with Bennett, Scha and van der Does in what I label the *quantificational* approach to numeral indefinites.

Of course, this is not the only way to pursue a modificational approach to numerals. For theories that do not take singular indefinites to be ordinary quantifiers there is another natural way to continue with plural indefinites. Probably the most elaborate treatment of plurals within a non-quantificational theory of

[4] There is a classical question whether to take (8) as equivalent to *at least three women smiled* (as Barwise & Cooper have it) or as equivalent to *exactly three women smiled*. Henceforth I adopt Barwise & Cooper's Gricean habit to attribute "precision" impressions in cases like (8) to a conversational implicature.

[5] This does not include certain versions of event semantics (cf. Schein (1993)).

[6] One common approach is to follow Bennett (1974) and Scha (1981) and to say that a plural individual corresponds to a set/predicate (i.e. it is of type et). Another popular line views collections as a special kind of "simple" individuals (of type e) as in Link (1983) or Landman (1989), which are related to their atomic parts in some way. To abstract away from this debate I will try to remain neutral with respect to this question for the sake of the intuitive discussion in this section and section 3. In the more formal sections 4-6 I will adopt some common assumptions that allow implementation in both ways.

[7] To name two examples, in the context of other determiners (e.g. in an NP like *the three girls*) or predicative constructions (e.g. *they are three*) numerals behave semantically more like "normal" adjectives, as a modificational approach leads one to expect. Compare the interpretation of these constructions to *the tall girl*, and *she is tall*, respectively.

indefinites is Kamp & Reyle (1993). In the modificational analysis of numerals that Kamp & Reyle develop the DRT representation of (9) is quite similar to (10), where the "plural individual" variable is a set variable. To deal with cases like (8), Kamp & Reyle introduce a distributivity operator that optionally applies in the process of predication. Roughly, a sentence like (8) can be represented in the same way as (9), plus a distributivity "mediator" D between the predicate and the set variable, as illustrated in (11). The notation $\mathbf{smile}' - D - (x)$ means that the predicate applies to every *singular* member of the plural individual x.

(11) $\exists x[\mathbf{three}'(x) \wedge \mathbf{women}'(x) \wedge \mathbf{smile}' - D - (x)]$

The introduction of a distributivity operator is very common in the literature on plurality and will be further discussed in subsection 3.3.1 below. Kamp & Reyle give a precise definition of this operator within the DRT framework.

We mentioned two problems of indefinites, the scope problem and the plurality problem, and considered some proposals for solution. It is natural to ask now: are there any interesting interactions between these problems? What kind of unified theory can account for them both? The next section proposes an answer.

3 A theory of indefinites and choice functions

3.1 On the structure of the argument

In this section I develop an argument in favour of a particular modification of Reinhart's proposal. I will adopt the choice function (CF) treatment in Reinhart's paper but will reject her proposal to treat distributivity of numeral indefinites using an additional strategy of generalized quantifiers (GQ). Instead, it is proposed that choice functions should be used as a *uniform* strategy for interpreting indefinites. Distributivity will be derived using an independent mechanism, required also for the interpretation of non-indefinite NP's.

The proposed account deals with simple singular indefinites like *a dog*, *some cat* and plural indefinites like *some dogs*, *several cats* and *three cows*. At present, these are the only kinds of indefinites that can be undoubtedly classified as "free scope takers". I henceforth sloppily refer to this group as "indefinites". Other indefinites like bare plurals (e.g. *dogs*), modified numerals (*exactly one cat*, *at most ten cows*), "vague" indefinites (*many/few people*) and indefinite partitives (*three of the men*) will not be discussed. The scope and plurality problems of these indefinites involve certain complex empirical and theoretical questions that deserve a separate study. [8]

The major part of the argument will concern the quantificational free scope approach and variations on the choice function approach. Certain important aspects of the Fodor & Sag/Kratzer specificity approach and the Ruys/Abusch representational approach will be discussed as we go along. However, I do not discuss these proposals in detail. My reasons for not doing so are methodological, and let me say a few words about them.

One disturbing aspect of the specificity idea is its approach to truth conditions. As far as I can understand, the specific reading of a sentence like *some woman smiled* is taken to be equivalent for certain speakers in certain contexts to the sentence *Princess Diana smiled*. What can we make of this factual claim? The acclaimed "pointing gesture within the mind of the speaker" is supposed to be a "private" one, so how can we *falsify* this idea? Since in many cases this is not clear to me at all, I cannot evaluate the adequacy of the specificity approach to indefinites. [9]

[8] See some remarks in subsection 3.5.

[9] For more elaborate doubts about the notion of specificity see Higginbotham (1987:64-66), Ludlow & Neale (1991), Ruys (1992:96-100).

My problem with the representational view of Ruys and Abusch is quite different and it has to do with the *restrictiveness* of their proposals. Both authors add special rules to their systems in order to deal with the scope of indefinites. These rules do not follow from any general property of the systems assumed. I therefore believe that if the liberal theoretical standpoint of Ruys and Abusch is seriously adopted it would allow us to account for *any* possible fact about the scope of indefinites by adding new rules or modifying existing ones. [10] In this situation, scientific argumentation is almost impossible because virtually *any* empirical argument you might come with can be refuted by your opponent, simply by postulating a new rule in her system. A point that concretely exemplifies this general criticism will be shown in subsection 3.3.3.

Note that by this objection I do not take any position in the debate about the necessity of a representational level as in DRT. [11] In fact, the indebtness of the choice function approach to the basic idea in DRT about the non-purely-quantificational nature of indefinites will be emphasized in the next subsection.

To facilitate the discussion we will assume a standard syntactic scope mechanism of Quantifier Raising (QR), restricted by island constraints. The null semantic hypothesis will be that NP's are treated as generalized quantifiers. QR semantically results in λ-abstraction over a variable at the NP's argument position, as in Montague's Quantifying-in operation. Thus, a sentence like (12) has an object wide scope (WS) reading that is represented at Logical Form (LF) as in (12a), which is translated into the proposition in (12b).

(12) Most women saw every man.

a. [every man]$_2$ [[most women]$_1$ [e_1 saw e_2]]

b. $[\lambda P.\mathbf{man}' \subseteq P]\ \ [\lambda x_2.[\lambda A.\ |A \cap \mathbf{woman}'|\ > |\overline{A} \cap \mathbf{woman}'|]\ \ (\lambda x_1.\mathbf{see}'(x_2)(x_1))]$

We thus standardly assume that a scope mechanism that is restricted at least by syntactic islands and has the semantic consequences of combining QR with a GQ analysis gives empirically correct results. Note that the possibility that the scope of some NP's is more restricted than others, as proposed in many recent works, is left open.

The rest of this section is organized as follows. My interpretation of Reinhart's choice function analysis is informally given in subsection 3.2. The main argument is developed in subsection 3.3 where a solution is proposed to the scope problem and the plurality problem. Subsection 3.4 argues for the proposed CF treatment of narrow and "intermediate" scope. Some remaining problems are mentioned in subsection 3.5. The overall picture is summarized in subsection 3.6.

3.2 Choice functions informally

Reinhart's treatment of indefinites using choice functions is based upon the following general assumptions:

(A1) Indefinites lack quantificational force of their own. They are basically *predicates*.

(A2) An indefinite NP in an argument position, however, ends up denoting *an individual*. This is because its semantics involves a free function variable that assigns an individual to the restriction predicate.

(A3) This function variable is existentially closed, together with the restriction that it is a choice function: a function that chooses a member from any non-empty predicate it gets. This *quantificational* procedure can apply at any compositional level.

[10] To do justice to Ruys's work, it must be said that he does not really consider his mechanism to be a semantic theory, but rather an "existence proof" of the possibility to interpret indefinites *in situ*. As said above, given his assumptions I think this attempt is bound to succeed.

[11] The closely related question of compositionality will become relevant only in section 5 of this paper.

Assumption (A1) is adopted from DRT. Familiar motivations for it are: (i) The natural appearance of indefinites in predicative positions (unlike, for example, singular NP's with *every*). (ii) The fact that many languages lack an overt indefinite article, which suggests that some phonologically invisible mechanism handles function application when the indefinite is in argument position. The idea is that the same happens in languages like English, where the overt indefinite article is devoid of denotational contribution. [12] (iii) The well-known discourse-anaphoric properties of indefinites.

Assumption (A2) is a specific proposal to deal with the type mismatch that (A1) creates. In general, the predicate denoting the N' of an NP in an argument position can either be lifted to a quantifier (the usual strategy) or lowered to an individual. The second strategy is the grammatical option assumed for indefinites. This idea can be traced back to Hilbert & Bernays (1939)'s ϵ-logic, which treats existentials as *terms*. [13]

Assumption (A3) is also borrowed from DRT, minus the "unselective binding" strategy, plus the stipulation about restricting existential closure to choice functions. [14]

Let us consider a simple concrete example:

(13) Some woman smiled.

By (A1)-(A3), what (13) is understood to assert is a statement about the existence of a choice function, such that the individual it picks from the predicate denoting *woman* is in the extension of the predicate denoting *smiled*. This intuition is given a formula in (14) below, where $CH(f)$ means that f is a choice function.

(14) $\exists f[CH(f) \wedge \mathbf{smile'}(f(\mathbf{woman'}))]$

Of course, in order for (14) to be well-defined, the CH condition should be defined explicitly. At least one part of the required definition for CH was mentioned above. Namely:

(15) **The choice condition**: A function f is a choice function (i.e. $CH(f)$ holds) *only if* for every non-empty predicate P, $f(P)$ is defined and it is in the extension of P (i.e. $P(f(P))$ holds).

This is the standard requirement in various definitions of choice functions in the literature (see e.g. Egli & von Heusinger (1995) for details). Here, the choice condition is deliberately stated only as a necessary condition and not as a sufficient one. In section 4 it will be claimed that the condition in (15) is actually not a sufficient condition for defining CH and a complete definition will be proposed. Keeping this in mind, (15) is both intuitive and adequate enough to serve the purposes of the informal discussion to follow in the present section. For the time being, let us assume that (14) is equivalent to the standard analysis of (13): $\exists x[\mathbf{woman'}(x) \wedge \mathbf{smile'}(x)]$. More content to this assumption will be given in section 4 below.

What Reinhart has shown is that an analysis along these lines intuitively accounts for the scope problem of indefinites. For example, sentence (7) has now two readings, depending on the stage at which existential closure is performed:

(16) $[\exists f[CH(f) \wedge \mathbf{come'}(f(\mathbf{woman'}))]] \rightarrow \mathbf{glad'}(\mathbf{j'})$

(17) $\exists f[CH(f) \wedge [\mathbf{come'}(f(\mathbf{woman'})) \rightarrow \mathbf{glad'}(\mathbf{j'})]]$

In (16) existential closure is within the antecedent of the conditional. Consequently, we get the "narrow scope" reading paraphrased in (16') below. In (17), existential closure takes scope over the conditional, and this results in a "wide scope" reading, paraphrased in (17').

[12] See arguments for a similar idea about conjunctive morphemes in Winter (1995a).

[13] Hilbert's program influenced instantial logics and logics of arbitrary objects. See Meyer Viol (1995) and Fine (1985) for extensive developments of these two perspectives. In linguistics the epsilon tradition has been persistently followed in works of the "Konstanz School" about anaphora and (in)definites (see e.g. Egli & von Heusinger (1995)).

[14] In the current state of the theory, this is a stipulation. However, in a work in progress I propose that "choice" follows from general universals on determiners in generalized quantifier theory.

(16') John will be glad if there is *any* possibility to pick a woman who comes to the party.

(17') There is a choice function such that John will be glad if *the woman it picks* comes to the party.

The innovative aspect of Reinhart's idea is that in both readings no mechanism "pulls" the representation of the indefinite out of the island: in both cases the relevant predication is analyzed as $\mathbf{come}'(f(\mathbf{woman}'))$.

3.3 The semantics and scope of numeral indefinites

In this subsection I develop the main linguistic argument of this paper. A preliminary distinction will be made between two abstract notions of distributivity: *quantificational* (Q) distributivity and *predicate* (P) distributivity. Then I will first elaborate on an observation from Ruys (1992) about the island-restricted behaviour of Q-distributivity. This observation is argued to show a major advantage of Reinhart's CF analysis over alternative proposals. However, a second observation to be made will show that Reinhart's mechanism, which is already quite complicated, is still empirically inadequate. I will introduce my modification of Reinhart's system and show how it explains the facts. Some remaining questions about the *standard* scope mechanism with indefinites will then be addressed.

3.3.1 Predicate Distributivity and Quantificational Distributivity

Before dealing with the plurality problem of indefinites, let us briefly discuss the issue of distributivity with general plural NP's, not necessarily indefinites. The term "distributivity" is often used to refer to the intuition that sentences like (18) and (19) are logically entailed by the universal statements in (18a) and (19a) respectively. [15]

(18) The boys ate.

a. *Every* boy ate.

(19) The boys ate a cake.

a. *Every* boy ate a (potentially different) cake.

I would like to distinguish between two mechanisms for modeling the effects of distributivity. This abstract distinction will help in the analysis of some concrete phenomena in the next subsections, without making a commitment at this stage to any particular theoretical proposal.

Consider first (18). Assume that **b**' denotes a "plural individual" (e.g. a set) corresponding to the collection of boys referred to in (18). Consider now the following question: is it *necessary* that the statement that (18) makes explicitly includes a universal quantification as in (20) in order to capture the entailment (18a)⇒(18)?

(20) $\forall x \in \mathbf{b}'[\mathbf{eat}'(x)]$

(21) $\mathbf{eat}'(\mathbf{b}')$

[15] This inference is sometimes referred to as *cumulative reference*. The other direction does not hold. Sometimes sentences like (18)/(19) are assumed to have a *distributive reading* which is equivalent to (18a)/(19a). Here I deliberately refrain from suggesting that this is the case *pre-theoretically*. Nevertheless, a standard account of distributivity as developed below eventually derives distributive readings as a theoretical construct.

Some theories of plurality (e.g. Scha (1981), Hoeksema (1983)) answer this question negatively. The reason is that also (21) can capture the distributivity effect, without directly assuming any explicit universal quantification. Instead, the distributivity impression we get in (18) might come from a meaning as in (21) in conjunction with the lexical semantics of the predicate *eat* (what Dowty (1986) calls the "sub-entailments" of the predicate). Thus, the entailment (18a)⇒(18) might be considered to reflect only a meaning postulate about the predicate *eat*: $\forall x[(\forall y \in x\ \mathbf{eat}'(y)) \rightarrow \mathbf{eat}'(x)]$, where x ranges over plural individuals. Such a line was taken by Scha and Hoeksema. Even though this might not be the analysis for (18) one eventually adopts, this is a logical possibility. We may refer to these cases of distributivity which can *in principle* be derived from some "distributivity property" of the *lexical* predicate by the term *Predicate* (P) *distributivity*. This term should not be confused with any theoretical device. Note especially that it refers only to a subset of the facts which Link (1983)'s familiar distributivity operator on predicates explains. This will be obvious from the discussion below.

Now, is (19) also a case of P-distributivity? That is: can (23), without any explicit universal quantification, guarantee the entailment (19a)⇒(19) using some lexical assumptions on the predicate?

(22) $\forall x \in \mathbf{b}'\ \exists y[\mathbf{cake}'(y) \wedge \mathbf{eat}'(y)(x)]$

(23) $\exists y[\mathbf{cake}'(y) \wedge \mathbf{eat}'(y)(\mathbf{b}')]$

Here the answer is negative. Even if we assume that the two place predicate $\mathbf{eat}'$ in (23) lexically "distributes" over the members of $\mathbf{b}'$ as we tentatively assumed for (21), this still does not reflect (22). The information such lexical information would add to (23) is only that it might be true in case *all* boys ate the same cake. But in a situation where there is only a *different* cake for each boy this proposition would still not be satisfied. The reason is that the existential quantification in (23) has "wider scope" than the predicate, hence also wider than any implicit universal quantifier triggered by the "lexical distributivity" of the verb. [16] We may conclude that (19) is not a case of P-distributivity: some quantificational mechanism should account for the entailment (19a)⇒(19). Let us refer to such cases by the term *Quantificational* (Q) *distributivity*. [17]

The same distinction can be made when the plural definite in (18)-(19) is replaced by a simple numeral indefinite as in (24)-(25).

(24) Three boys ate.

a. There is a set of three boys, such that *each* member of this set ate.

(25) Three boys ate a cake.

a. There is a set of three boys, such that *each* member of this set ate a (potentially different) cake.

[16] See also van der Does (1992:32-33) on the same point. Things are somewhat different if we let transitive predicates have quantifiers as their arguments, as in classical Montague semantics. Technically, this strategy allows one to give the distributivity operator in the predicate scope over its quantifier argument. For the sake of exposition, I do not consider this complex possibility. However, even this complication does not seem sufficient, because a plural NP can distribute over expressions which are not co-arguments of the same predicate. For example, in *the boys will be glad if a girl comes*, the matrix sentence's subject can distribute over the conditional, i.e. every boy will be glad if a (potentially different) girl comes.

[17] Note that Q-distributivity appears also when the object is not a simple indefinite. Similar effects appear also with the following sentences: (i) is true in the situation described in (i_a) and (ii) in the situation described in (ii_a).

(i) The boys ate less than three courses.

a. John ate only the first and the second course, Bill ate only the appetizer and the desert, etc.

(ii) The boys ate most courses.

a. Among the three courses served: John ate the first and the second courses, Bill ate the first and the third, etc.

Let us first concentrate on the phenomenon of Q-distributivity, which will be of primary interest in what follows. How do semantic theories account for this effect? The following two answers are among the most popular ones:

- *Generalized Quantifiers* (GQ) and ambiguity of determiners: As mentioned above, for the standard treatment of NP's as generalized quantifiers over singular individuals, the problem of how to get Q-distributivity as in (19)/(25) does not arise at all. However, to get collectivity as in (27) below, a distributive/collective ambiguity of determiners was used in the "pure GQ" treatments of Bennett, Scha and van der Does.

- *Operational Distributivity* (OD): In many theories of plurality an operator of distributivity is introduced. Often it is proposed that this operator applies to an NP denoting a plural individual, shifting it into a universal quantifier over singular individuals (see for example Heim, Lasnik & May (1991)). [18] Consequently, (19)/(25) can be represented as in (26), which is interpreted as (22)/(25a).

  (26) $[\![\text{the/three boys}]\!]^D$ $[\![\text{ate a cake}]\!]$

Once Q-distributivity is correctly captured by a GQ/OD mechanism, one might suspect that all cases of P-distributivity are also captured by it. However, most theories of plurality assume that plural individuals can appear as direct arguments of "distributive" predicates like *to eat* (see (21)) in the same way as they are required to appear as arguments of a "collective" predicate:

(27) The boys met.

a. $\mathbf{meet}'(\mathbf{b}')$

There are some good reasons to allow a reading parallel to (27a), as was given in (21), also for (18). Two of them follow:

1. Distributivity of predicates is not an absolute notion. [19] A predicate like *to eat* is not completely distributive in cases like:

   (28) The boys ate the cake.

   (28) might be true even if not *every* boy ate the cake. Thus, an obligatory Q-distributivity treatment here is too strong. We might wish to allow (28) to be true in a situation where some boy didn't eat the cake but was a member of the collective of boys that ate it. This can be achieved by representing the sentence as $\mathbf{eat}'(\mathbf{b}')(\mathbf{c}')$, analogous to (21).

2. Complex predicates like *meet and eat* should apply to plural individuals for allowing collectivity in the first conjunct. But then it is technically hard to assume that the argument of the second predicate is not a plural individual.

Let us summarize. We follow the common assumption in cases of collective predication the denotation of the NP ranges over plural individuals. All predicates can take plural individuals as their arguments. In principle, this allows us to treat cases of P-distributivity as well. However, deriving Q-distributivity requires either an additional denotation of plural NP's using GQ's over singular individuals or an implicit distributivity operator (OD). One of the claims below will be that the latter strategy is preferable.

[18] Other common practices of OD locate an operator of distributivity on the predicate or in the composition process of the predicate with its NP argument. As van der Does (1992:32-33) notes, these treatments must allow OD also on *non-lexical* predicates (e.g. the complex predicate *ate a cake*) in order to be able to derive effects of Q-distributivity. Once this assumption is made explicit there is no empirical difference between these approaches to OD that is relevant to the discussion.

[19] See Dowty (1986:p.103) for a lucid discussion.

3.3.2 The Ruys observation and its significance

Both Ruys (1992) and Abusch (1994) observe that the "scope" of plural numeral indefinites can violate islands, quite like singular indefinites. This is to be expected. However, Ruys has observed one surprising fact about the scopal behaviour of indefinites. To see the point, consider one of Ruys's more simple examples: [20]

(29) If three relatives of mine die I will inherit a house.

The WS reading of the plural indefinite over the conditional in (29) can be attested by the following situation. Suppose I have three old uncles: Paul, George and Ringo who own together a beautiful house in England. Each of them owns one third of the house and plans to bequeath it to me. Now (29) might well be true even though I also have many poor relatives who will not bequeath me anything when they pass away. This means (29) has a reading where the plural "takes scope" over the conditional. But what exactly is this reading? The two possible candidates are paraphrased in (30) and (31).

(30) I have three relatives such that if they (all) die I will inherit a house.

(31) I have three relatives such that for *each* of them, if he dies I will inherit a house.

Ruys's point is that (30) captures the wide scope reading of (29) whereas (31) does not. Consider the situation in which uncle Paul is dead and uncle George and uncle Ringo are alive and well. I go to the court and ask to inherit the house. The court rejects my claim, so I have no chance to inherit the house as long as uncles George and Ringo are alive. Does this situation falsify (29)? Certainly not. It also does not falsify (30). However, (31) is falsified. Conclusion- (30) is the correct paraphrase of the wide scope reading of (29).

What does this mean? The difference between (30) and (31) is with respect to the scope of Q-distributivity. (30) is either a case of P-distributivity (predication over the collection of relatives) or a case where Q-distributivity scope is within the conditional. Which possibility among the two is the correct analysis does not matter for us at this stage because they both lead to identical truth conditions. However, in paraphrase (31) Q-distributivity takes scope over the conditional which leads to inadequacy.

Tentatively, we can say that with respect to *existential scope*, (29) behaves quite like the singular indefinite case (7). However, with respect to *Q-distributivity scope*, (29) behaves more like (5), disallowing a scope position out of the island.

The above distinction involves some semantic intuitions that might seem subtle. However, the fact that Q-distributivity does not appear with island-escaping scope can be shown to be highly robust. A way to test this is to construe an example in which the only pragmatically plausible reading would constitute an island-escaping case of Q-distributivity. If the sentence turns out to be odd, it must mean that the reading in question is not there. A relevant test is the following variation on Ruys's example:

(32) # If three women gave birth to John then he has a nice mother.

a. Existential narrow scope + P-distributivity (=Q-distributivity narrow scope):
   # If (each member of) any set of three women gave birth to John, then he has a nice mother.

b. Existential wide scope + P-distributivity(=Q-distributivity narrow scope):
   # There is a set A of three women such that if (each member of) A gave birth to John then he has a nice mother.

[20] This particular example is from Ruys (1995). Ruys (1992) contains many other examples to support what I henceforth call the Ruys observation.

c. Existential wide scope + Q-distributivity wide scope:
   * There is a set A of three women such that for each member x of A, if x gave birth to John then he has a nice mother.

If Q-distributivity beyond island boundaries were possible, (32) would have been expected to have a reading as paraphrased in (32c), which expresses a contingent and completely reasonable proposition given common world knowledge about the predicate *to give birth*. However, we can be sure that (32c) is not a reading of (32), because (32) is clearly a strange thing to say: it implies the possibility that three women *all* gave birth to John. The existential narrow scope construal for (32) in (32a) is pragmatically odd in the same way. So is the WS existential reading in (32b). The fact that the sentence sounds so odd is therefore to be expected if Q-distributivity beyond islands is not an available option.

Similar effects appear also with other islands, as shown in the following examples:

(33) # Every artist who was born in three cities became famous. [21]

a. * There is a set A of three cities such that for each member x of A, every artist who was born in x became famous.

(34) # A baby was adopted by John or was born to three women.

a. * There is a set A of three women such that for each member x of A, a baby was adopted by John or was born to x.

In (33), wide scope for Q-distributivity beyond the Complex NP island as paraphrased in (33a) would eliminate the oddness effect. For similar reasons, (34a) is surely not a reading of (34), where the plural indefinite is within a Coordinate Structure island.

Let us summarize the empirical conclusion:

The Ruys observation: While the *existential* scopal semantics of indefinites can violate island constraints, the scope of *Q-distributivity* is island-restricted. [22]

What is the theoretical significance of the Ruys observation? Ruys's work suggests that it necessitates a serious complication in the "free scope" approach to indefinites. The LF representation of (29) according to this approach is as in (35). An analysis of (35) using GQ's (following Bennett, Scha, or other quantificational

[21] Some similar examples given in Abusch (1994) marginally allow Q-distributivity scope beyond the island. Crucially, all the examples Abusch gives (her (42)-(49)) involve only one CNPC island. (i) is another example for such a marginal WS Q-distributivity effect. However, the same island-violating effect also arises with a *universal* NP in this construction, as in (ii).

(i) ? Every child who was born to three famous women became famous too.

(ii) ? Every child who was born to every famous woman became famous too.

Thus, cases like (i) or Abusch's sentences are not counter-examples to the Ruys observation, but only an indication that one CNPC barrier might marginally allow *standard* scope beyond it, also for non-indefinites. A similar point was mentioned to me by Shalom Lappin (p.c.) using the example in (iii).

(iii) A delegate who was elected from each district was disqualified.

(iv) A minister who knows a delegate who was elected from each district was disqualified.

This might have to do with Chomsky (1986)'s relativist notion of barrierhood: crossing one barrier is sometimes "easier" than crossing two. For example, in (iv) a wide scope reading for the universal NP beyond two CNPC islands is completely impossible. The point relevant for the discussion above is that cases like (i) or the Abusch sentences, to the extent that they allow island-escaping scope for Q-distributivity, are not an appropriate test for the main argument developed in this subsection.

[22] The facts discovered by Ruys were not originally described in these terms. However, Ruys's work is the source of this generalization. It seems that Schein (1993:pp.205-6 and fn.35) suggests a similar generalization.

approaches to the plurality problem) must block the distributive reading of the "island escaping" indefinite. But a collective reading must still be allowed in order to capture the WS reading.

(35) $[\![$three relatives$]\!]\ \lambda x.[\![$if x die I will inherit a house$]\!]$

How can we block the distributive reading? As far as I know, only by means of stipulation – by giving the NP some feature that will disallow a distributive reading if the standard scope mechanism assigns it wide scope beyond an island. Such an unattractive assumption is additional to the stipulation inherent to the "free scope" approach about indefinites: now we have also a sub-class of readings for indefinites that *are* sensitive to island constraints. Namely, distributive readings. Unattractive as it is, in the next subsection we will see that even this additional stipulation cannot save the "free scope" approach.

Reinhart's approach to plural indefinites shows some improvement. According to Reinhart, there are two ways to represent numeral indefinites: one using distributive GQ's and another using CF's. There is no independent distributivity mechanism. [23] The standard scope mechanism of QR does not assign indefinites syntactic scope beyond islands, so Reinhart does not need to block any reading of (35) because in her proposal this is not an available LF of (29). Reinhart's CF reading of the indefinite *in situ* with wide scope existential closure takes care of the "island-escaping" effect:

(36) $\exists f[CH(f) \wedge [\mathbf{die}'(f([\![\text{three relatives}]\!])) \rightarrow \mathbf{inherit\_house}']]$
"There is a choice of a plural individual, x, consisting of three relatives of mine, such that if x dies I will inherit a house."

By definition, choice functions apply only to predicates. Therefore, crucially, in order for (36) to be meaningful, the denotation $[\![$*three relatives*$]\!]$ must be a predicate over plural individuals: the set of all plural individuals consisting of exactly three women. The choice function picks one plural individual from this set. To do that, Reinhart adopts the modificational analysis of *three*, as in the other "numerals as adjectives" proposals mentioned above. Reinhart is not explicit about how to derive now also the alternative distributive GQ denotation she assumes. I think this is a weak aspect of her proposal, which will be modified in what follows. For the time being, note that (36) does not involve any overt distribution of the plural individual chosen, and for this reason it generates a wide scope *existential* reading without any Q-distributivity effect. Rather, lexical properties of the predicate are considered responsible for the P-distributivity effect. This is sufficient to capture the Ruys observation. Let us consider another example for this, not discussed by Reinhart, which shows another effect discovered in Ruys (1992:102, ex.(36)):

(37) Some student who followed two courses in Mathematics I heard of is a genius.

The curious effect in such examples with two indefinites is that a "wide scope" reading for the plural indefinite does not appear at all. This is of course completely unexpected in the free scope approach without the further stipulation mentioned. However, nothing about (37) is surprising for the CF approach. The sentence gets the following CF analysis: [24]

(38) $\exists g \exists f[CH(f) \wedge CH(g) \wedge \mathbf{genius}'(f(\mathbf{student}' \cap \lambda x.\mathbf{follow}'((g([\![\text{two courses}]\!])))(x)))]$

The order of the existential quantifications over choice functions ($\exists g \exists f$ or $\exists f \exists g$) does not affect the proposition expressed by (38). Essentially for this reason, this proposition correctly reflects the meaning of (37): the NP *some student* cannot be understood within the scope of a distributive reading of *two courses*. The

[23] This is the original proposal in the prepublished version of Reinhart (1996). Taking into account the problems pointed out in the present paper, Reinhart (1996) is less confident about the possibility to eliminate operational distributivity.

[24] We assume also other possible lower positions for the existential closure of the CF g variable. However, for the "WS" reading of the plural only (38) is relevant.

latter indefinite gets only *collective* wide scope (+ P-distributivity), which in this case is not distinguished from collective narrow scope.

I consider these predictions of Reinhart's mechanism to be an insightful account of the Ruys observation. But we must answer the question of how to derive now Q-distributivity, to make sure that the mechanism indeed works. We are up to discover another surprising fact.

3.3.3 The double scope observation and its significance

Reinhart adopts what I called the GQ approach to Q-distributivity. We have considered already the problem for the "free scope" GQ approach to indefinites. Reinhart's system is not less stipulative in this respect: the only motivation for her GQ construal of the indefinite is the need to generate Q-distributivity with numeral indefinites. Moreover, in Reinhart's proposal it is not clear how to account for Q-distributivity with non-indefinite plurals (cf. *the boys* in (19)).

I will show now that the problems are not only conceptual. Both the "free scope" approach and Reinhart's proposal are still empirically insufficient. Consider the following example: [25]

(39) If three workers in our staff have a baby soon we will have to face some hard organizational problems.

This sentence certainly has a narrow scope reading for the indefinite. In this reading it claims that *any* three workers who will have a baby soon will cause problems to the organization. However, there is also a wide scope reading here, which can be paraphrased as follows:

(40) There are three workers such that if *each* of them has a baby soon we will have to face some hard organizational problems.

It is important to note is that this is *the only* reasonable reading of (39) where the plural indefinite "takes scope" over the conditional. To see this, consider the following alternatives:

(41) There are three workers, such that for *each* of them, if he/she has a baby soon we will have to face some hard organizational problems.

(42) # There are three workers, such that if there is a baby that they *all* have soon, we will have to face some hard organizational problems.

The Ruys observation tells us that (41) is not a reading of (39). To verify that, one has to use the same considerations we already applied in the analysis of (29) and (32). (42) is perhaps a grammatical reading of (39), but it is pragmatically implausible: it implies that three people can have one and the same baby, a situation most of us cannot imagine. In more theoretical terms, (39) shows a case in which the plural indefinite can take existential wide scope *beyond* the island, whereas a Q-distributivity effect appears, but its scope is *within* the island. Note that with respect to distributivity, this is the same effect we discussed concerning (19) and (25), so P-distributivity is excluded here.

Let us describe this effect as follows:

The double scope observation: The existential and the Q-distributivity imports of numeral indefinites can have two distinct scope positions.

Another example for this observation is the following sentence.

[25] The kind of effect that is exemplified here was discovered together with Eddy Ruys.

(43) Every teacher who gave a lecture to three excellent classes in our school was very satisfied afterwards.

a. There are three excellent classes in our school, such that every teacher who gave a (potentially different) lecture to them was very satisfied afterwards.

Also in (43), the existential scope of the plural indefinite can escape the island as in the (43a) paraphrase, where the possibility that each of the three classes was given a different lecture indicates that this is a case of Q-distributivity (having scope within the complex NP island).

According to the Ruys observation, when the indefinite is within an island Q-distributivity scope is restricted. Therefore, double scope is easily visible. Reasonably, however, numeral indefinites should be able to show "double scope" behaviour in any syntactic environment, independently of the Ruys observation. This can be attested in the following sentences, where the plural indefinites are not locked in islands.

(44) Exactly one teacher gave a lecture to three excellent classes in our school.

(45) Every teacher of three excellent classes in our school that were given a lecture was satisfied.

Consider for example the two existential-WS readings of the plural in (44), with the two possible scopes for Q-distributivity ($\exists!x$ stands for "exactly one x"):

(46) $\exists A[\mathbf{classes'}(A) \wedge \mathbf{three'}(A) \wedge \exists!x[\mathbf{teacher'}(x) \wedge \forall z{\in}A\ \exists y[\mathbf{lecture'}(y) \wedge \mathbf{gave'}(y)(z)(x)]]]$

(47) $\exists A[\mathbf{classes'}(A) \wedge \mathbf{three'}(A) \wedge \forall z{\in}A\ \exists!x[\mathbf{teacher'}(x) \wedge \exists y[\mathbf{lecture'}(y) \wedge \mathbf{gave'}(y)(z)(x)]]]$

Assume there are only three classes: c_1, c_2 and c_3. Teacher t_1 is the only teacher who gave a lecture to the three of them. Teacher t_2 also gave a lecture, but only to c_1. In this situation, (44) can intuitively be interpreted as true, like (46), whereas (47) is false. Thus the "double scope" reading (46) is attested for (44). A similar story goes for (45).

The double scope observation is a major problem for any pure quantificational approach to indefinites. Especially, it is a problem for the quantificational free scope approach. For example, in this approach (39) can have only one LF that derives the existential wide scope:

(48) [three workers]$_1$ [[if e_1 have a baby] we will face problems]

If the indefinite in (48) is analyzed as a distributive GQ, we overgenerate the non-reading (41). If the GQ is collective, we get the pragmatically implausible reading (42). We have only one position for both existential and Q-distributivity scope and therefore the reading (40) cannot be generated at all.

Reinhart's original proposal is faced with a similar problem. The CF representation can capture only (42). Reinhart does not allow any Q-distributivity effect in the CF "island-escaping" reading and consequently a double scope effect as in (40) is not generated. But *cannot* it be generated in the CF approach to the scope problem?

Here is my proposal. Let us readopt the common assumption about a grammatical operation of distributivity. To be concrete, I assume Heim, Lasnik & May's version of the operator, located on the NP. [26] Now, instead of Reinhart's CF/GQ double strategy we can let choice functions be the *uniform* mechanism for interpreting indefinites: GQ's are not necessary anymore for modeling Q-distributivity. The separation between the existential scope, derived by the CF mechanism, and the Q-distributivity scope, derived by OD, allows us to capture the double scope effect in (39) as follows:

[26] This is not an arbitrary preference. In Winter (1996b) I show that some operation turning a plural individual into a universal quantifier is advantageous for a non-ambiguous analysis of NP conjunction in *collective* contexts. However, as mentioned above, only the assumption about *some* operation of distributivity is relevant for us here, and not its exact formulation.

(49) $\exists f[CH(f) \wedge [((f([\![\text{three workers}]\!]))^D(\lambda x.\exists g(CH(g) \wedge \mathbf{have}'(g(\mathbf{baby}'))(x)))) \rightarrow \mathbf{problems}']]$

Let us examine this analysis in detail. *Three* is analyzed as an adjective. The indefinite *three workers* is therefore basically analyzed as a predicate: the set of plural individuals consisting of exactly three singular workers. The CF variable f picks one plural individual from this set. The distributivity operator D applies to this individual, to generate a universal quantifier over singular individuals. This quantifier is assumed to take scope over the predicate $\mathbf{have}'$, like any quantifier. [27] However, the scope of this quantifier (or the NP it denotes) is restricted to remain within the adjunct island, so it does not take scope over the conditional. The indefinite *a baby* is analyzed using a narrow scope existential closure of CF, assumed above in (A3). This representation intuitively captures reading (40) of (39). As will be shown in the following section, this is the case also formally.

After capturing the double scope observation we must be careful to guarantee that the proposed CF+OD mechanism also respects the Ruys observation. Namely, that Q-distributivity scope is still island-restricted. In fact, this is so because of a basic insight of Reinhart's approach, independent of our implementation of Q-distributivity. In Reinhart's CF treatment the restriction of the indefinite is never "pulled out" of a syntactic island. The universal quantifier derived by a distributivity operation is generated at the position where the restriction is. Hence the scope of Q-distributivity cannot violate islands. To get a feeling of how strong this prediction is, the reader is urged to challenge Reinhart's "don't pull restrictions out of islands" strategy by supplying her system with his/her preferred mechanism of distributivity so that Q-distributivity scope beyond islands *is* generated. This is not a trivial task. [28]

I think this robustness is a beautiful property of Reinhart's proposal. In case the Ruys observation turns out to be wrong it would be very hard to save the CF approach! This is in distinction to the Ruys/Abusch representational approach to the scope problem. For example, in Abusch's system, where the restriction is simply "pulled out", the operator of distributivity can be implemented to be island-restricted (as in Kamp & Reyle's system [29]), but it can also be designed to violate islands (as Abusch seminally proposes). This means that Abusch's system is less restrictive than Reinhart's with respect to the scope of Q-distributivity – whatever the facts are, Abusch's representational strategy can deal with them.

One additional remark. In criticizing Reinhart's GQ approach to the Q-distributivity of indefinites I certainly do not say that the GQ perspective is inadequate for the semantic analysis of *other* NP's. For instance, singular universal NP's have always been natural candidates for a treatment as distributive generalized quantifiers, and there seems to be no reason to change this view. In general, which effects of distributivity are to be accounted for using standard GQ's and which using non-lexical OD is a broad semantic question that goes beyond the scope of the present enterprise. [30]

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[27] In subsection 3.3.4 it will be claimed that indefinites should get standard restricted scope, like other NP's. In section 4 the individual yielded by the CF will be mapped up in a generalized quantifier, which is another motivation for this assumption.

[28] To do that, I tried for example to invent a notion of a "distributive choice function": a CF that generates a distributive universal quantifier *directly* from the restriction of the indefinite (by composing D to the output of a "normal" CF). Even then, the distributive quantifier is still generated at the same position of the restriction.

[29] Distribution in Kamp & Reyle's proposal is in the process of predication. Consequently, it does not escape adjunct islands to take scope over conditionals, for example.

[30] The case of plural pronouns is another example for NP's where an OD analysis as in Heim, Lasnik & May (1991) seems to be advantageous. In a simple sentence like (i) the easiest way of representing the Q-distributivity reading of the sentence proceeds by letting an operator of distributivity apply to the plural individual corresponding to the pronoun.

(i) They ate a cake.

A GQ analysis for such cases of Q-distributivity would require the plural pronoun to represent a variable over *quantifiers*. Such an analysis does not seem to be able to account for the fact that plural pronouns can be also interpreted as variables bound by a quantifier, i.e. variables over individuals. For example:

(ii) Most people smile to every baby they see.

Thus, an OD approach to Q-distributivity of plural pronouns is a natural way to analyze plural pronouns uniformly as variables over

3.3.4 Standard scope within island boundaries

We have seen that the proposed CF+OD approach has certain advantages over Reinhart's CF+GQ proposal. However, in one respect both strategies are too weak, at least with the explicit assumptions mentioned above. Consider for example the following sentence from Reinhart (1996) [31]:

(50) An American flag was hanging in front of two buildings.

a. Hanging in front of two buildings there was a (potentially different) American flag.

In order to get the prominent reading (50a) for (50), both approaches must assume that the standard scope mechanism applies to the indefinite NP object. In the GQ approach this is well-known. As regards to the CF treatment, we have seen in the analysis of (37) that a plural indefinite cannot Q-distribute over a singular indefinite that is not within its syntactic scope. The same point applies to (50).

Reinhart (1996) allows indefinites, like other NP's, to take standard scope restricted by islands. In an LF analysis this means that after QR applies, (50) has roughly the structure in (51), to which both CF+GQ and CF+OD assign the desired Q-distributive reading.

(51) [two buildings]$_2$ [[an American flag]$_1$ [e_1 is hanging in front of e_2]]

In general, such Q-distributive WS readings of plural indefinites appear in many cases where world knowledge makes the NS reading incoherent. This is the case also when the subject is not a simple singular indefinite. For example:

(52) Exactly one cat is sitting in five baskets.

(53) The mayor was disqualified in nine cities.

(54) Two guards are standing in front of twenty buildings in this town.

Such facts suggest that although Q-distributivity wide scope is not very common with indefinites, it is an existing possibility, which indicates that indefinites can take standard scope within island boundaries. Theoretically, this is of course highly plausible. Once a standard scope mechanism is used for other NP's there is no reason a priori to let indefinites be exempt from the operation of this mechanism. However, this disagrees with some proposals (e.g. Ruys (1992), Beghelli (1993), Ben-Shalom (1993)) that suggest that indefinites (among other NP's [32]) do not get syntactic wide scope. The factual motivation for the "no-syntactic-scope" approach is that plural indefinites are much less easily interpreted as having wide Q-distributivity scope than the classical case of singular *every*/*each* NP's. Consider for example the paradigm example of WS interpretation, that of "inverse linking":

(55) Some girl from every city is happy.

(56) #? Some girl from three cities is happy.

While a WS reading for (55) is prominent, a parallel reading for (56) is marginal, if existent at all. However, in that respect the plural indefinite in (56) behaves just like any other plural. For example:

(singular/plural) individuals. Cf. also Kamp & Reyle (1993:ch.4) for a detailed discussion of relevant points.

[31] The importance of such cases was recently exemplified in Fox (1995)'s insightful analysis of quantifier scope in VP ellipsis.

[32] Ruys, Beghelli and Ben-Shalom respectively propose that weak, non-distributive and non-principal-filter-denoting NP's do not take scope using the standard scope mechanism. As (57) shows, Ruys's classification is too narrow, as well as Ben-Shalom's. Beghelli's generalization is just not too clear, since his notion of distributivity is undefined. However, if one reads "distributive" as "headed by *every* or *each*" this is perhaps a sound description, but as indicated above, with a relative rather than an absolute sense: many plural NP's do show WS behaviour when put in object position, cf. (50)-(54).

(57) #? Some girl from all/these/most cities is happy.

There is no particular difference between (56) and (57) as regards to the availability of the WS reading for the plural NP's. The plural NP's in (57) are as strong as the *every* NP in (55) by any criterion of "strength" (e.g. ungrammaticality in *there* sentences or other definiteness effects, semantic definitions as in Barwise & Cooper (1981), Keenan (1987)). Thus, if the lack of a WS reading for the indefinite in (56) is to be accounted for in terms of its weakness or indefiniteness, it is unclear why also the strong NP's in (57) lack a WS reading.

Moreover, we cannot draw a sharp borderline by simply claiming that inverse linking contexts never allow WS for plural NP's. Availability of the WS reading for the plural increases significantly in cases like (58) and (59).

(58) A principal actor in ninety movies in the festival was American.

(59) A principal actor in all movies in the festival was American.

To summarize, the question to ask is not: why do plural *indefinites* refuse to get Q-distributive WS readings? Rather, the question is: why is it so *hard* for NP's other than *every*/*each* NP's (weak and strong alike) to get a Q-distributive WS reading? I have no answer to this question. The only point of the above discussion is that a theory that wishes to account for cases like (50)-(54) should assume standard scope for indefinites, restricted by island constraints. Theoretically, this is a very natural assumption. Empirically, further constraints on the standard scope of indefinites seem to stem from general restrictions on the scope of a much larger class of NP's, so they do not raise a major question that a theory about the scope of indefinites should answer. Needless to say, this is an intriguing question in its own right.

3.4 Narrow and intermediate scope

Assumption (A3), adopted from Reinhart's conception, contains a crucial aspect of my proposal. If choice functions, as I propose, are the only strategy for interpreting indefinites then they are responsible also for scopes narrower than the widest. Existential closure of CF variables can apply at any stage of the compositional derivation. Conceptually, this is the null hypothesis: an operation that "looks at the tree" and applies existential closure only at the highest matrix level would have to be superimposed on the CF mechanism. However, Kratzer (1995) disagrees with that. For Kratzer, because of the "referential" notion she adopts, even if there is quantification over choice functions, it takes only widest scope. Let me therefore show that non-widest existential closure of choice functions has independent motivation. The first argument is new. The second is a reassessment of arguments by Ruys, Abusch and Reinhart.

3.4.1 Coordinations with indefinites

In Winter (1996a-b) I proposed a way to analyze NP coordinations in a uniform way using only the standard Boolean representation for the coordinator *and*. A central argument in favour of this proposal is the possibility it opens for analyzing conjunction as unambiguous, in distinction to most theories of conjunction. For collective predication as in (60) it is often assumed that the NP conjunction should be analyzed using a non-Boolean lexical entry for *and*, in addition to the standard Boolean entry that is needed for the correct analysis of many other conjunctions, such as the predicate conjunction in (61).

(60) John and Bill met.

(61) Mary is tall and thin.

Without reviewing in detail the technical semantic mechanism in the Boolean treatment of cases like (60), we may summarize its main characteristics briefly as follows: (i) NP's are standardly treated as GQ's. (ii) Conjunction is standardly "intersective" (Boolean). (iii) A collectivity operator C turns a Boolean conjunction of GQ's into a plural individual set [33], to which the collective predicate applies.

For example, the coordinate subject NP of (60) is standardly treated in terms of GQ intersection, i.e. (62a). The conjoined GQ is transformed into a plural (set) individual using the C operator, as illustrated in (62b). The collective predicate applies to this plurality as in (62c).

(62) a. $\{A \mid \mathbf{j}' \in A\} \cap \{A \mid \mathbf{b}' \in A\} \;=\; \{A \mid \mathbf{j}' \in A \wedge \mathbf{b}' \in A\}$
b. $\mathsf{C}(\{A \mid \mathbf{j}' \in A \wedge \mathbf{b}' \in A\}) \;=\; \{\mathbf{j}', \mathbf{b}'\}$
c. $\mathbf{meet}'(\{\mathbf{j}', \mathbf{b}'\})$

Arguments for the advantages of this treatment over some other alternatives were given at length in Winter (1996a). Here I will simply assume this treatment, on which the argument below is based.

Given the proposed analysis of conjunction, some serious problems arise for the standard quantificational analysis of indefinites. One example from Winter (1996a) is the case of simple NP coordinations like *John and some man*. In the GQ approach, this NP is analyzed as in (63). If we take John to be a man, then the denotation of the NP coordination is the same GQ as the one that denotes the proper name *John*. Intuitively, the reason is that any set containing John trivially contains both John and a man (namely, John himself). The opposite direction is also trivial.

(63) $[\![\text{John and some man}]\!] \;=\; \{A \mid \mathbf{j}' \in A\} \cap \{A \mid \exists x[x \in A \wedge \mathbf{man}'(x)]\}$
$\mathbf{man}'(\mathbf{j}') \quad \Rightarrow \quad [\![\text{John and some man}]\!] \;=\; \{A \mid \mathbf{j}' \in A\} \;=\; [\![\mathrm{John}]\!]$

So, if indefinites are compositionally analyzed as GQ's and if conjunction is intersective, then there is no semantic difference between (64) and (65). [34] This is of course incorrect.

(64) John and some man met.

(65) *? John met.

In Winter (1996a) I showed that this problem disappears once indefinites are treated as in the DRT tradition, viz. as free variables existentially bound at some higher level in the sentence. The same holds for the CF analysis of indefinites: when the indefinite *some man* is analyzed as an individual chosen by the CF variable, (64) gets essentially the same treatment as (60) got in (62). Namely:

(66) $\exists f[CH(f) \wedge \mathbf{meet}'(\mathsf{C}(\{A \mid \mathbf{j}' \in A \;\wedge\; f(\mathbf{man}') \in A\}))]$

There is a CF, f, such that the the collectivized Boolean coordination of the GQ's corresponding to John and to the man picked by f is a plural individual that met.

[33] In fact, this description is oversimplified. The proposed C operator generates a higher order quantifier (over sets). See Winter (1996b) for a general motivation to use the operators composing C within flexible semantics. Also, as shown in that paper, the set-theoretical analysis of plurals in Winter (1996a) can be translated to other algebraic systems.

[34] Gennaro Chierchia and Danny Fox (p.c.) mention the possibility that in (64) the set denoted by *man* excludes John because of some domain restriction. In Winter (1996a) I argue against this approach. A more simple argument than the ones mentioned in that paper comes from plural indefinites. The domain restriction approach predicts (i) to be false in case three authors who are also teachers are in the room. This is a problematic prediction.

(i) Three authors and three teachers are in the room.

In other words, since the indefinite *some man* is treated as an individual it gets the same analysis the GQ for *Bill* gets in (60). Existential quantification over CF's guarantees that a choice of such a man exists. Thus, (66) should reflect correctly the meaning of (64). Note that it can well happen in (66) that for some value of f we get $f(\mathbf{man'}) = \mathbf{j}'$. In this case we get the false proposition $\mathbf{meet'}(\{\mathbf{j}'\})$. However, because of the existential quantification over CF's this does not *have to* happen: (66) intuitively means that there is a man who met John, just like (64).

The crucial point for our discussion here is that if we adopt the analysis of (64) along the lines of (66), then we can construct a direct argument in favour of existential closure of CF variables at lower levels than the matrix sentence level. Consider the following example:

(67) Every piano was lifted by John and some man.

(67) can be true in the following situation S: piano p_1 was lifted by John and Bill *together*, piano p_2 was lifted by John and George together, and so on, for all pianos. This means that the NP *John and some man* should have a "collective" interpretation of a plural individual, similar to the analysis of (64). However, it was argued above that in the adopted analysis of conjunction this is not possible with a GQ representation for the indefinite; we must use the CF analysis. On the other hand, (67) is true in S, where different pianos are lifted by John with *different* men. Thus, the existential import of the indefinite *some man* in (67) should be assigned narrower scope than the scope of the universal quantification in the sentence. That is, we should allow (67) to have the representation in (68), where $\{\mathbf{j}', f(\mathbf{man'})\}$ is derived using C and Boolean coordination of GQ's as in (66).

(68) $\forall x[\mathbf{piano'}(x) \rightarrow \exists f[CH(f) \wedge \mathbf{lifted\_by'}(\{\mathbf{j}, f(\mathbf{man'})\})(x)]]$

The same point can be exemplified with any transitive verb that allows a collective interpretation of its object argument. Consider for example:

(69) Every girl defeated John and some boy.

(70) Every group surrounded this skyscraper and some building.

(71) Every dish was eaten by Mary and some girl.

3.4.2 Intermediate scope

A topic that received much attention in Fodor & Sag (1982), Farkas (1981), Ruys (1992:109-115) and Abusch (1994) is the question whether indefinites can have island escaping scope without having wide<u>st</u> sentential scope. Fodor & Sag claimed that this possibility does not appear. However, Farkas, Ruys and Abusch, among others have shown some cases that challenge Fodor & Sag's generalization. For example:

(72) Every professor will rejoice if a student of his cheats on the exam. (Ruys (1992))

a. For every professor there is a student of his such that if this student cheats the professor will rejoice.

The entailment (72a)⇒(72) shows that the scope of the indefinite need not be sentential when it escapes the island: it can be narrower than the scope of the subject of the matrix sentence, but wider than the island. This kind of readings is often referred to as *Intermediate Scope* (IS).

Kratzer (1995) revives Fodor & Sag's "referential" approach, according to which IS is not generally an available option. Kratzer claims that the pronoun within the indefinite NP in cases like (72) is the source of the IS reading. [35] Her argument is based on the observation that when the bound pronoun is replaced by a deictically interpreted expression, as in (73), then the IS reading seems to disappear, in a similar way to Fodor & Sag's original example (74).

(73) Every professor will rejoice if a student of mine cheats on the exam. (Kratzer (1995))

(74) Each teacher overheard the rumor that a student of mine had been called before the dean. (Fodor & Sag (1982))

Anticipating such an argument, Reinhart (1996) decisively shows (as claimed before by Farkas) that the pronoun is not necessary for an IS reading. Reinhart's example (slightly changed) is the following:

(75) Every linguist has looked at every analysis that solves some problem.

a. IS: For every linguist x there is a problem y such that x has looked at every analysis that solves y.

(75a) is an available reading for (75): in a situation in which every linguist has her preferred problem, and she has looked at every analysis that solves it (75) is true. In fact, (75a) is the pragmatically *prominent* reading for (75): assuming (somewhat optimistically) that every analysis solves some problem, the NS reading of (75) reduces to (75b), and the WS reading is (75c). Both readings are reasonably false, as perhaps most readers can personally verify.

(75) b. NS: Every linguist has looked at every analysis.

c. WS: There is a problem x, such that every linguist has looked at every analysis of x.

Once we have identified the reasoning behind Reinhart's construal of the example, we can duplicate the IS effect with many other sentences. For example:

(76) Every movie director is happy to direct every film that features some actor.

a. IS: For every director x there is an actor y such that x is happy to direct every film featuring y.

(77) Every country's security will be threatened if some building is attacked by terrorists.

a. IS: For every country there is a (potentially different) building, an attack on it might threaten the country's security.

Also in these cases, the IS reading is the prominent one. The principle in constructing these examples is to reduce the plausibility of all readings but the IS reading. This point reveals a methodological weakness in the Fodor & Sag/ Kratzer choice of examples. Sentences (73)-(74) have both plausible NS and WS readings for the indefinite. However, when claiming that some interpretative strategy does not exist one should concentrate on cases in which this strategy is likely to be pragmatically preferred. Ideally, one should choose examples where the reading in question is *the only* coherent reading. Since (73)-(74) are not such cases, they are problematic test cases. Pragmatic factors can have a confounding effect that contributes to the determination of which readings appear prominent when such a multitude of interpretations is present. The fact that one of the readings is not prominent is a very weak piece of evidence. Thus, the Farkas/Ruys/Abusch/Reinhart argument is decisive: intermediate scope readings of indefinites are there.

For the CF theory of indefinites, this is another indication that non-widest-scope existential closure should be available. It has to be possible in order to represent (75), to consider only one case of IS, as in (78). The analysis applies analogously to the other cases.

(78) $\forall x[\mathbf{linguist}'(x) \rightarrow \exists f[CH(f) \wedge \forall y[(\mathbf{analysis}'(y) \wedge \mathbf{solve}'(f(\mathbf{problem}'))(y)) \rightarrow \mathbf{look\_at}'(y)(x)]]]$

[35] Kratzer does not mention Farkas (1981), where the examples used to argue against Fodor & Sag do not involve anaphora.

3.5 Problems and loose ends

As mentioned above, there is a large group of indefinites about which the present account has nothing to say. Firstly, as exemplified already in Carlson (1977), *bare plurals* show an exceptionally *narrow* scope behaviour. Carlson concluded that ordinary existential quantification is problematic for treating bare plurals. This conclusion applies also to the CF analysis. In the case of *modified* numerals and *few*/*many* indefinites even the correct statement of the scope and plurality facts is not completely clear. For example: is there a WS reading for the indefinite in (79)? What is precisely the "collective" statement expressed by (80)?

(79) If less than four relatives of mine die I will inherit a house.

(80) Less than four relatives of mine gathered.

There are some answers proposed to these questions in the literature (see Beghelli (1993), van der Does (1992), Reinhart (1996), Ruys (1992), Schein (1993), among others) but as far as I can see, no one of them is fully decisive. Of course, as stressed in van Benthem (1986:51-54), any treatment of non-upward monotone NP's as predicates, plus some operation of existential quantification, is highly problematic. The CF analysis is no exception. However, it is far from being clear that CF's should be used in these cases for any reason.

There are also certain poorly understood phenomena of differences between the singular articles *a* and *some* with respect to wide scope availability and the exact function of expressions like (*a*) *certain*. Another question is whether the choice function theory of indefinites has any interesting ramifications for a theory of *definites*.

The discussion of some other complex problems in the domain of *intensionality* and *anaphora* is deferred to subsection 4.6. For the time being, let us conclude this section with a summary of what has been proposed.

3.6 Summary

The main claims made in this section are:

- The CF approach is a conceptually plausible method to analyze the semantics of simple indefinites and it derives their free scopal behaviour.
- Simple plural numerals are modifiers. The CF approach, plus an operation of distributivity, correctly captures the distinction between island-escaping existential scope of numeral indefinites and their island-restricted Q-distributivity scope. The GQ approach to distributivity (e.g. Bennett, van der Does, Scha, Reinhart) does not make this distinction.
- The standard scope mechanism applies to indefinite NP's in the same way as it applies to non-indefinites. Further restrictions that apply to the scope of NP's other than the *every*/*each* classical wide scope takers might affect also the standard scope of indefinites.
- The mechanism of CF is not responsible for widest existential scope alone. It is the uniform strategy for the interpretation of indefinites.

The linguistic study of the CF mechanism justifies a deeper investigation of the formal system required for an explicit implementation of it. This is the topic of the second part of this paper.

4 Aspects of formalization

The preceding discussion was based on the hope that the assumed mechanism of choice functions can be given a precise semantic definition that correctly describes the behaviour of indefinites in natural languages. The aim of the present section is to provide formal support for this hope. We will address a basic problem for a theory that treats indefinites as individuals – the interpretation of indefinites with an empty restriction set. A semantic solution will emerge from a negative observation about the most common approach to choice functions. It will be proved that in the standard semantics of the typed lambda calculus, this approach cannot correctly handle the empty restriction problem. An alternative formalization will be proposed and application of this formalism to the informal analyses in section 3 will be shown to lead to a correct treatment of representative examples.

4.1 The empty restriction problem

Reconsider (13), repeated as (81) below.

(81) Some woman smiled.

A typed formula as in (82), which was freely used above, is the most straightforward way to analyze (81) in the CF approach.

(82) $\exists f_{(et)e}[CH(f) \wedge \mathbf{smile}'_{et}(f(\mathbf{woman}'_{et}))]$

For (82) to be well-defined, we need of course to define the predicate CH over $(et)e$ functions. One necessary condition for such a definition was given in (15). The commonly assumed definition of choice functions is simply a strengthening of this condition into a definition, as given in (83).

(83) $CH \stackrel{def}{=} \lambda f_{(et)e}.\forall P_{et}[P \neq \emptyset \rightarrow P(f(P))]$ [36] (a temporary definition)

Using this definition we can derive the following equivalence for (82):

(84) $\exists f[CH(f) \wedge \mathbf{smile}'(f(\mathbf{woman}'))] \Leftrightarrow$
$\exists f[\forall P_{et}[P \neq \emptyset \rightarrow P(f(P))] \wedge \mathbf{smile}'(f(\mathbf{woman}'))] \Leftrightarrow$
$\exists x[\mathbf{woman}'(x) \wedge \mathbf{smile}'(x)] \vee [\neg\exists x\, \mathbf{woman}'(x) \wedge \exists x\, \mathbf{smile}'(x)]$

This equivalence shows an undesired result: when the set of women is empty, (83) allows a choice function f to choose *any* individual. Thus, if in some situation the set of women turns out to be empty, and some individual smiles, then there is a function in CH that assigns this individual to the empty set. Consequently (82) becomes *true*.

This result is clearly problematic. Consider (85), in which the restriction is more likely to be empty than in (81). No speaker who knows that the USA is a republic would accept (85) as a coherent and truthful utterance. The proposition generated by the analysis sketched above, however, would be true if Mary met some individual.

(85) Mary met an American king.

We may conclude that such a straightforward approach to choice functions is at best linguistically incomplete. It does not account for (85) in the present political state of affairs in America, where the restriction of the indefinite is an empty set. I henceforth refer to this problem as *the empty restriction problem*.

[36] I use the notation $\emptyset_{\tau t} \stackrel{def}{=} \lambda X_\tau.\bot$ for every type τ.

4.2 An inexpressibility problem

The problem we are facing is the assignment of the the non-standard reading (84) to (81), instead of the standard existential quantification (86).

(86) $\exists x[\mathbf{smile'}(x) \wedge \mathbf{woman'}(x)]$

The choice function mechanism should better make (81) equivalent to (86) without further complications. More generally, we look for a definition of the predicate $CH_{((et)e)t}$ that makes (i) and (ii) equivalent for any predicates A,B.

(i) $\exists f_{(et)e}[CH(f) \wedge A_{et}(f(B_{et}))]$

(ii) $\exists x_e[A(x) \wedge B(x)]$

One may think that the failure to do so in subsection 4.1 is only due to the specific definition given to choice functions in (83). However, there is *no* definition for CH that can render (i) and (ii) equivalent using the standard syntax and semantics of the typed lambda calculus. To prove that, let us observe the following, more general fact:

Proposition 4.1 *Let $D_{(et)((et)t)}$ be a conservative determiner. Then:*
$[\exists CH_{((et)e)t}\, \forall A_{et}\, \forall B_{et}\, [\, \exists f_{(et)e}[CH(f) \wedge A(f(B))] \leftrightarrow D(B)(A)\,]\,] \rightarrow \forall A\, \forall B\, [\neg D(B)(A)]$

Proof*: See appendix A.*

This proposition claims that a formula (i) can be equivalent to $D(B)(A)$ only if the determiner D non-conservative or trivial (does not hold of any A and B). In particular, we get the following corollary:

Corollary 4.2 $[\exists CH_{((et)e)t}\, \forall A_{et}\, \forall B_{et}\, [\, \exists f_{(et)e}[CH(f) \wedge A(f(B))] \leftrightarrow \exists x_e[A(x) \wedge B(x)]\,]\,] \rightarrow \forall x \bot$

Proof*: Follows from conservativity of the existential determiner $\mathbf{E} = \lambda X.\lambda Y.\exists z[X(z) \wedge Y(z)]$ and the biimplication $\forall A \forall B[\neg \mathbf{E}(B)(A)] \leftrightarrow \forall x \bot$*

This negative result shows that under standard logical assumptions we cannot expect a formula in the form of (i) to be equivalent to the existential quantification in (ii). [37] In order to do that we have either to change the standard semantics of (i) or to somehow change the syntax of (i). The first possibility is unattractive– changing logical semantic mechanisms is not the first thing one would like to do upon encountering an empirical linguistic problem. Fortunately, there is a simple way to change slightly the form of (i) so that equivalence between (i) and (ii) is retained.

4.3 A solution: back to Boolean individuals

I would like to suggest that the problem is a problem of typing: it arises because the straightforward analysis in (i) treats the subject of predication (the expression $f(B)$) as an individual in the non-Boolean domain D_e. However, a fruitful alternative approach to individuals, following Montague, is to let them live in the Boolean $(et)t$ domain of *quantifiers*. An individual a_e is mapped to the set of its properties: $\lambda P.P(a)$.

[37] Importantly, note that this result does not show that it is impossible to reflect (ii) with another logical formula that uses choice functions. For example: the proposition $\exists f_{(et)e}[CH(f) \wedge A(f(B)) \wedge B(f(A))]$ *is* equivalent to (ii), according to definition (83). Actually, the result in Meyer Viol (1995:25) shows that *any* proposition of first-order logic is expressible in ϵ-logic (whose semantics as given by Meyer Viol is an extension of the definition in (83)). However, this fact is of little help for a theory interested also in the syntax of natural language sentences: it is not clear how to compositionally obtain such a CF/ϵ-logic formula *directly from LF* (unless LF is a completely arbitrary representation with little respect to surface structure). The existence of some translation procedure from Predicate Calculus/Generalized Quantifier format to ϵ format is hardly an answer to this question.

[38] Let us assume that a choice function, instead of choosing an e-type individual from a non-empty set, chooses the $(et)t$ quantifier corresponding to it. Thus, choice functions are of type $(et)((et)t)$: the type of *determiners*. Consequently, the typing of (i) changes as follows:

(i') $\exists f_{(et)((et)t)}[CH(f) \wedge (f(B))(A)]$

Note that the change of types entails a structural change in the formula, because the function-argument relations between $f(B)$ and A reverse. Logically, the Montagovian treatment of individuals as GQ's is well-motivated. Linguistically, this is also entirely natural – we retain a quantificational type for the indefinite NP, which is now type-theoretically treated like any quantificational NP or type-lifted individual. This is congenial also in view of the motivation for a unified scopal treatment of indefinites and other NP's that was given in subsection 3.3.4.

How can CH be defined to guarantee that (i') is equivalent to (ii)? With the new typing this is rather straightforward:

- When B is not empty, a choice function assigns it the generalized quantifier corresponding to some individual in B.
- When B is empty, a choice function chooses *the empty quantifier*– the trivial quantifier which does not include any set of individuals.

The first condition has the desired effect of definition (83) when B is not empty. The second condition guarantees that when B is empty, $(f(B))(A)$ is *false* since $f(B)$ is empty. Formally, this definition reads as follows:

(87) **The choice definition**:

$$CH \stackrel{def}{=} \lambda f_{(et)((et)t)}.\forall P_{et}[P \neq \emptyset \rightarrow \exists x_e(P(x) \ \wedge \ f(P) = \lambda A_{et}.A(x))] \ \wedge \ f(\emptyset_{et}) = \emptyset_{(et)t}$$

With this definition it is not hard to establish the following:

Fact: (i') $\Leftrightarrow$ (ii)

Thus, definition (87) overcomes the problem of definition (83): simple sentences like (81) are now treated using choice functions in a way equivalent to standard existential quantification. This correct treatment is of course not a significant semantic achievement in its own right. Things become more interesting, however, because now we can check the adequacy of the proposed definition for a wider range of facts. In the rest of this section I will show that definition (87) fares well also with the more complex data of section 3.

4.4 Examples- singular indefinites

Let us henceforth assume definition (87). The first point to note about this definition is its special treatment of the scope effects of indefinites. From the large literature on NP "scope" one is sometimes led to think schematically of a "wide scope" interpretation for an indefinite as existential quantification with the restriction predicate "pulled out". The CF approach, however, treats the indefinite "locally", without moving the restriction syntactically. This difference is roughly illustrated in (88a-b).

(88) ... some woman ...

[38] In Boolean terms, this is the principal ultrafilter generated by a. Keenan & Faltz (1985) even propose to eliminate the domain D_e of individuals from the ontology.

a. $\exists x[\mathbf{woman}'(x) \wedge \ \dots\,]$

b. $\exists f[\ \dots\ f(\mathbf{woman}')\ \dots\,]$

In the case of (81) we saw that definition (87) prevents any undesired semantic difference between the two approaches. However, when potential scope relations between the indefinite and other elements in the sentence exist, differences show up. Consider first (89).

(89) Every artist who was born in some city became famous.

(90) $\exists x[\mathbf{city}'(x) \wedge \forall y[(\mathbf{artist}'(y) \wedge \mathbf{born\_in}'(x)(y)) \rightarrow \mathbf{famous}'(y)]]$

(91) $\exists f[CH(f) \wedge \forall x[(\mathbf{artist}'(x) \wedge f(\mathbf{city}')(\lambda y.\mathbf{born\_in}'(y)(x))) \rightarrow \mathbf{famous}'(x)]] \ \Leftrightarrow$
$[\exists x[\mathbf{city}'(x) \wedge \forall y[(\mathbf{artist}'(y) \wedge \mathbf{born\_in}'(x)(y)) \rightarrow \mathbf{famous}'(y)]]] \ \vee \ \neg\exists x \ \mathbf{city}'(x)$

(90) is usually assumed to express the wide scope reading of the indefinite in (89). (91) is the corresponding CF representation. These two formulae do not reflect the same proposition: in a situation where the restriction predicate $\mathbf{city}'$ is not empty, (91) gets the same value as (90). However, when this predicate is empty, (90) is *false* whereas (91) is *true*. Intuitively, (91) gets this value because $f(\mathbf{city}')$ is then empty for any choice function. Therefore the restriction of the universal in (91) is false, so any choice function can satisfy (91). But, is it reasonable to model (89) as *true* when no cities exist?

The answer is positive. As far as the *narrow scope* reading of (89) concerns, this is the standard truth value assigned to the sentence. Theoretically, however, the *wide scope* reading standardly assigned to the sentence is *false* in the situation mentioned above. But, in practice direct linguistic facts do not easily provide evidence for this treatment. The reason is that effects coming from the empty restriction of the indefinite interact here with effects of the empty restriction of the universal NP. To exemplify the problem, let us contrast the following sentences:

(92) Every artist who was born in Antarctica became famous.

(93) Every artist who was born in some city in Antarctica became famous.

Assuming that no artist was born in Antarctica and that there is no city in Antarctica, there is no semantic contrast between (93) and (92): both seem equally “odd”. However, standardly, (92) is treated as true. I would like to claim that for this reason it is hard to determine what truth value a reading of (93)/(89) should get when the restriction of the indefinite is empty. It is almost impossible to decide whether the oddness of (93) comes with a false statement due to the empty restriction *of the indefinite* in a “wide scope reading” like (90) or is it oddness resulting of an empty restriction *of the universal NP* in a “choice function reading” like (91), which standardly has a *true* semantic value obtained in a way similar to (92). Semantically, there is a difference between the two representations (90) and (91), but in practice it is hard to decide which one is correct.

However, one might reasonably want to look for other cases that supply more robust evidence deciding between the standardly assumed WS reading and the CF reading generated by definition (87). One such possible case is the following:

(94) John feeds a certain brave rabbit or milks a certain red cow.

a. WS: $\exists x\exists y[\mathbf{brave\_rabbit}'(x) \wedge \mathbf{red\_cow}'(y) \wedge [\mathbf{feed}'(x)(\mathbf{j}') \vee \mathbf{milk}'(y)(\mathbf{j}')]]$

b. CF: $\exists f\exists g[CH(f) \wedge CH(g) \wedge$
$[f(\mathbf{brave\_rabbit}')(\lambda x.\mathbf{feed}'(x)(\mathbf{j}')) \ \vee \ f(\mathbf{red\_cow}')(\lambda y.\mathbf{milk}'(y)(\mathbf{j}'))]] \ \Leftrightarrow$
$\exists x[\mathbf{brave\_rabbit}'(x) \wedge \mathbf{feed}'(x)(\mathbf{j}')] \vee \exists y[\mathbf{red\_cow}'(x) \wedge \mathbf{milk}'(y)(\mathbf{j}')]$

The "free scope" assumption about indefinites implies that (94) has the "wide scope" reading (94a), where the scope of both indefinites violates the coordinate structure island. According to the CF mechanism, (94b) is the correct "WS" interpretation. The crucial point is that (94) does not assert the existence of *both* brave rabbits and red cows as (94a) does. Existence of only one of these rare kinds of animals should be enough for (94) to be true, in case John takes care of the creature. Here the CF formula (94b) is preferable: it reduces to sentential disjunction as in (95), which intuitively paraphrases (94).

(95) John feeds a brave rabbit or John milks a red cow.

In other cases studied, the predictions of definition (87) are very close to the standard WS reading, in a way that is similar to the different predictions discussed with respect to (89).

Below a couple of examples are given, with the calculation of the (acceptable) predictions of definition (87).

(96) If some woman comes to the party John will be glad. ((7) repeated)

a. WS: $\exists f[CH(f) \wedge [f(\mathbf{woman'})(\mathbf{come'}) \rightarrow \mathbf{glad'}(\mathbf{j'})]] \Leftrightarrow$
$[\exists x[\mathbf{woman'}(x) \wedge [\mathbf{come'}(x) \rightarrow \mathbf{glad'}(\mathbf{j'})]]] \vee \neg\exists x\ \mathbf{woman'}(x)$

(97) Every linguist has looked at every analysis that solves some problem. ((75) repeated)

a. IS: $\forall x[\mathbf{linguist'}(x) \rightarrow \exists f[CH(f) \wedge \forall y[(\mathbf{analysis'}(y) \wedge f(\mathbf{problem'})(\lambda z.\mathbf{solve'}(z)(y))) \rightarrow \mathbf{look\_at'}(y)(x)]]] \Leftrightarrow$
$\forall x[\mathbf{linguist'}(x) \rightarrow [\exists z[\mathbf{problem'}(z) \wedge \forall y[(\mathbf{analysis'}(y) \wedge \mathbf{solve'}(z)(y)) \rightarrow \mathbf{look\_at'}(y)(x)]]]] \vee$
$\neg\exists z\ \mathbf{problem'}(z)$

4.5 Examples- plural indefinites

To exemplify the formal treatment of plural indefinites using choice functions we should allow reference to plural individuals. Theories of plurality usually adopt the following assumptions about the domain E of individuals:

- $\mathcal{A} \subseteq E$ is the set of *atomic* individuals.
- $\Pi \subseteq \mathcal{A} \times (E \setminus \mathcal{A})$ is *the atomic part-of relation*: $\Pi(a)(x)$ holds iff a is an atom that is a part of the plural individual x.

In Link's formalization of plurality, the type of the domain E is e and Π is determined by a certain algebraic structuring of $\mathcal{A}$ into E. In the Bennett/Scha set theoretical approach, E is given the type et and $\mathcal{A}$ and Π are determined by the standard Boolean definitions. [39] To avoid complex typing, I henceforth use the type e for E. However, I will deliberately remain vague about the algebraic structure of E (the exact definition of $\mathcal{A}$ and Π), in order to allow also a straightforward set theoretical translation of the formalism.

In Heim, Lasnik & May (1991) (among others), the distributivity operator D' is defined as follows:

For every $x \in E \setminus \mathcal{A}$: $D'_{e((et)t)}(x) \stackrel{def}{=} \lambda P_{et}.\forall y_e \in \mathcal{A}[\Pi(y)(x) \rightarrow P(y)]$ [40]

[39] $\mathcal{A} = \{\{x\} \in D_{et} | x \in D_e\}$ (the Boolean atoms of D_{et})

$\Pi(X)(Y)$ holds iff $X \subseteq Y$ (Boolean ordering of D_{et})

[40] The restriction $y \in \mathcal{A}$ is redundant and used only for clarity.

A generalized quantifier formulation, which is more convenient when individuals are lifted to the $(et)t$ domain, is the following: [41]

$$D_{((et)t)((et)t)} \stackrel{def}{=} \lambda Q_{(et)t}.\begin{cases} D'(x) & \text{if there is a unique element } x \text{ is } E \setminus \mathcal{A} \text{ s.t. } Q = \lambda P_{et}.P(x) \\ Q & \text{otherwise} \end{cases}$$

That is, principal ultrafilters generated by non-atomic individuals are mapped to the corresponding distributive quantifier. Other quantifiers are left untouched.

Plural nominals are standardly treated as the set of plural individuals composed of singular individuals in the corresponding set of singular individuals. For example:

$$\mathbf{boys}'_{et} \stackrel{def}{=} \lambda x_e.\forall y_e \in \mathcal{A}[\Pi(y)(x) \to \mathbf{boy}'(y)] \ \wedge \ |\{z|\Pi(z)(x)\}| \geq 2$$

Numerals too have a standard definition as modifiers. For example:

$$\mathbf{three}'_{(et)(et)} \stackrel{def}{=} \lambda P_{et}.\lambda x_e.P(x) \wedge |\{y|\Pi(y)(x)\}| = 3$$

The above assumptions are to be expected in any theory of plurality that, like the theory argued for in section 3, adopts the approaches of "operational distributivity" and "numerals as modifiers". To see how this mechanism works for an elementary example, reconsider (25):

(98) Three boys ate a cake.

The object narrow scope construal involves two representations: (98a), where the subject denotation is not distributed, and (98b), where it is distributed using D.

(98) a. $\exists f[CH(f) \wedge f(\mathbf{three}'(\mathbf{boys}'))(\lambda x.\exists g[CH(g) \wedge g(\mathbf{cake}')(\lambda y.\mathbf{eat}'(y)(x))])] \Leftrightarrow$
$\exists x[\mathbf{three}'(\mathbf{boys}')(x) \wedge \exists y(\mathbf{cake}'(y) \wedge \mathbf{eat}'(y)(x))]$

b. $\exists f[CH(f) \wedge D(f(\mathbf{three}'(\mathbf{boys}')))(\lambda x.\exists g[CH(g) \wedge g(\mathbf{cake}')(\lambda y.\mathbf{eat}'(y)(x))])] \Leftrightarrow$ [42]
$\exists x[\mathbf{three}'(\mathbf{boys}')(x) \wedge \forall z \in \mathcal{A}[\Pi(z)(x) \to \exists y(\mathbf{cake}'(y) \wedge \mathbf{eat}'(y)(z))]]$

(98a) is true iff there is a certain cake that a collection of three boys ate. (98b) is true iff there is some collection of three boys such that each member of it ate a cake by himself.

The above definitions allow a correct treatment of the scope and distributivity effects of plural indefinites that were exemplified in section 3. This is exemplified by the analysis of the WS reading of the plural indefinites in (99) and (39), restated here as (100).

[41] The definition is given for convenience in a condition format. More formally, the definition reads as follows:
$\mathrm{NAG}_{((et)t)(et)} \stackrel{def}{=} \lambda Q.\lambda x.x \in E \setminus \mathcal{A} \ \wedge \ Q = \lambda B.B(x)$ (x is the Non-Atomic Generator of Q)
And then: $D \stackrel{def}{=} \lambda Q.\lambda P.\exists x[\mathrm{NAG}(Q)(x) \wedge D'(x)(P)] \ \vee \ [[\neg\exists x \mathrm{NAG}(Q)(x)] \wedge Q(P)]$

This is not the common practice of defining the D operator for NP's. But since it is assumed here, following Montague and Keenan & Faltz (1985), that individuals are lifted to the $(et)t$ domain, some revision in the standard $e((et)t)$ D' operator is required. However, the above strategy is not the only plausible one and can be replaced by one of the following: (i) We can use the M^{-1} (Montague lowering) operator from Partee (1987) to derive e-type individuals from principal ultrafilters; (ii) We can allow CF's to also yield output of type e, using a disjunctive type for their range. I do not consider these technical possibilities here.

[42] *Sketch of proof*: Assume $\mathbf{three}'(\mathbf{boys}') \neq \emptyset$. Then for every f, $CH(f) \to f(\mathbf{three}'(\mathbf{boys}')) = \lambda P.P(x)$, for some $x \in \mathbf{three}'(\mathbf{boys}')$, by definition of CH. $x \in E \setminus \mathcal{A}$ by definition of $\mathbf{three}'$ and $\mathbf{boys}'$. Thus, we get:
$\exists f[CH(f) \wedge \exists x[f(\mathbf{three}'(\mathbf{boys}')) = \lambda P.P(x) \wedge \forall z \in \mathcal{A}[\Pi(z)(x) \to \exists g[CH(g) \wedge g(\mathbf{cake}')(\lambda y.\mathbf{eat}'(y)(z))]]]]$, that is:
$\exists x[\mathbf{three}'(\mathbf{boys}')(x) \wedge \forall z \in A[\Pi(z)(x) \to \exists y(\mathbf{cake}'(y) \wedge \mathbf{eat}'(y)(z))]]$

Assume $\mathbf{three}'(\mathbf{boys}') = \emptyset$. Then $CH(f) \to f(\mathbf{three}'(\mathbf{boys}')) = \emptyset_{(et)t}$ and by definition of D we get:
$\exists f[CH(f) \wedge \emptyset_{(et)t}(\lambda x.\exists g[CH(g) \wedge g(\mathbf{cake}')(\lambda y.\mathbf{eat}'(y)(x))])] \ \leftrightarrow \ \perp$

(99) Every student who followed two courses in Mathematics I heard of must be a genius.

a. $\exists f[CH(f) \wedge \forall x[(\mathbf{student'}(x) \wedge D(f(\mathbf{two'}(\mathbf{courses'})))(\lambda y.\mathbf{follow'}(y)(x))) \rightarrow \mathbf{genius'}(x)]] \Leftrightarrow$
$\exists y[\mathbf{two'}(\mathbf{courses'})(y) \wedge \forall x[(\mathbf{student'}(x) \wedge \forall z \in \mathcal{A}(\Pi(z)(y) \rightarrow \mathbf{follow'}(z)(x))) \rightarrow \mathbf{genius'}(x)]] \vee$
$\neg \exists y[\mathbf{two'}(\mathbf{courses'})(y)]$

(100) If three workers in our staff have a baby soon we will have to face some hard organizational problems.

a. $\exists f[CH(f) \wedge [(D(f(\mathbf{three'}(\mathbf{workers'})))(\lambda x.\exists g[CH(g) \wedge g(\mathbf{baby'})(\lambda y.\mathbf{have'}(y)(x))])) \rightarrow$
$\mathbf{problems'_t}]] \Leftrightarrow$
$\exists x[\mathbf{three'}(\mathbf{workers'})(x) \wedge [[\forall z \in \mathcal{A}[\Pi(z)(x) \wedge \exists y(\mathbf{baby'}(y) \wedge \mathbf{have'}(y)(z))]] \rightarrow \mathbf{problems'}]] \vee$
$\neg \exists x[\mathbf{three'}(\mathbf{workers'})(x)]$

Recapitulating the ideas in this section, it has been claimed that the object picked up by a choice function is not an e-type individual but rather a quantifier. Thus, the grammar provides determinerless indefinites with the missing determiner. This is the choice function variable whose free existential closure with the correct CF semantics accounts for the free existential scope of indefinites.

4.6 A note on intensionality and anaphora

So far, we have considered choice functions in extensional contexts only. In intensional contexts as in (i)-(ii), the CF analysis with WS existential closure is expected to provide the *de re* reading of the indefinite.

(i) John believes that some cat snored.

(ii) If John believes that some cat snored then he is probably mistaken.

Sentence (ii) exemplifies more clearly the urgency of the matter. The standard scope mechanism is normally used in such cases to derive the *de re* reading equivalent to: *there is a cat such that if John believes that it snored then he is probably mistaken*. If, as we claim, any reading of the indefinite in (ii) with scope over the conditional cannot be generated by the standard scope mechanism, then we must guarantee that the CF mechanism correctly derives this reading.

Let us concentrate on (i), a correct treatment of which using CF's should naturally extend also to more complicated cases like (ii). Reinhart (1996) proposes a treatment of intensional choice functions. The definition relies on letting the CF apply to the *property* denoting the restriction, deriving an individual in the *extension* of this property. Thus, an *intensional choice function* (ICF) is of type $(s(et))e$ and the set of ICF's is defined as follows:

(iii) $ICH^R \stackrel{def}{=} \lambda f_{(s(et))e}.\forall P_{s(et)}[\check{}P \neq \emptyset_{et} \rightarrow [\check{}P](f(P))]$

Using this definition, the *de re* reading of (i) is analyzed by:

(iv) $\exists f[ICH^R(f) \wedge \mathbf{bel'}_{(st)(et)}(\hat{}[\mathbf{snore'}(f(\hat{}\mathbf{cat'}))])(\mathbf{j'})]$

In case the extension of $\mathbf{cat'}$ is not empty, this means that there is a function attributing the property $\hat{}\mathbf{cat'}$ a member x of its extension such that John believes that x snored. This is the intended *de re* reading in the non-empty case. I leave here the empty case incorrectly treated. [43]

[43] A natural resolution is to use precisely the same strategy employed above with extensional CF's: lifting the output of the CF into the quantificational type. Here we are talking about an intensional quantifier of type $(s(et))t$ (Bennett's typing): a set of properties. The general type of ICF's is therefore $(s(et))((s(et))t)$ and their revised definition is as follows:

Such technicalities are necessary for a possible world treatment of CF's in intensional contexts. However, there is a problem that is noticeable in the extensional treatment of CF's even without going to classical *de re* contexts. This has to do with the interactions between CF's and "bound variable" anaphora. Consider sentence (v) with a subject-bound reading for the pronoun.

(v) *Every man* loves a woman *he* knows.

Given our extensional analysis, this sentence can get a "wide scope reading" for the indefinite as in (vi), while the pronoun is still bound by its antecedent.

(vi) * $\exists f_{(et)((et)t)}[CH(f) \wedge \forall x[\mathbf{man}'(x) \rightarrow f(\lambda y.\mathbf{woman}'(y) \wedge \mathbf{know}'(y)(x))(\lambda z.\mathbf{love}'(z)(x))]]$

This interpretation is wrong. To see that, assume that John and Bill are the only men and that they both know exactly the same women: Mary and Sue. Suppose further that John loves (only) Mary and Bill loves (only) Sue. Sentence (v) is intuitively true. However, (vi) turns out false. This formula requires that there is a CF f such that every man, x, loves the individual f assigns to the predicate $\lambda y.\mathbf{woman}'(y) \wedge \mathbf{know}'(y)(x)$. For the John and Bill entities instantiated for x this is precisely the same predicate. Therefore, f must assign it the same individual. Consequently, (vi) requires that John and Bill love the same woman. But, unlike the intuitive value of (v), this statement is *false* in the situation just described.

Roughly speaking, the origin of the problem is that the extensional CF is "insensitive" to the existence of a free variable in its argument. Intensionalising CF's might be a way to resolve this problem. Let us demonstrate it using Reinhart's definition (iii), according to which (v) is analyzed as follows:

(vii) $\exists f_{(s(et))e}[ICH^R(f) \wedge \forall x[\mathbf{man}'(x) \rightarrow \mathbf{love}'(f(\hat{}[\lambda y.\mathbf{woman}'(y) \wedge \mathbf{know}'(y)(x)]))(x)]]$

Now, the *property* $\hat{}[\lambda y.\mathbf{woman}'(y) \wedge \mathbf{know}'(y)(x)]$ *can* vary with different values assigned to x even in the situation described above. Specifically, when John and Bill both know only Sue and Mary in the actual world, this does not preclude that their acquaintances with women might be different in other possible worlds. Consequently, the ICF can assign the two different properties of being a woman John/Bill knows two different women in the actual world. The problem is thus avoided.

Needless to say, a more substantial investigation of the interactions between intensionality, anaphora and choice functions is definitely required. The above remarks only point to one possible direction. An open question is whether the complex strategy of Kratzer (1995), who proposes to use full-fledged Skolem functions in cases like (v) is well-motivated, given that an independently needed treatment of intensionality seems to solve the problem. The account in Chierchia (1993) of "list readings" of questions using Skolem functions is relevant for evaluation of this point as well.

In the next section we move, however, to a different problem of choice function interpretation.

5 Aspects of compositional formalization

In this section we will focus on the following question: what compositional process for interpreting syntactic structures can accommodate the proposed choice function analysis? This question is technically and

(1) $ICH \stackrel{def}{=} \lambda f_{(s(et))((s(et))t)}.\forall P_{s(et)}[[\check{}P \neq \emptyset_{et} \rightarrow \exists x_e([\check{}P](x) \wedge f(P) = \lambda A_{s(et)}.[\check{}A](x))]$
$\wedge[\check{}P = \emptyset_{et} \rightarrow f(P) = \emptyset_{(s(et))t}]]$

Due to the change of types, the analysis of (i) changes as follows:

(2) $\exists f[ICH(f) \wedge \mathbf{bel}'(\hat{}[f(\hat{}\mathbf{cat}')(\hat{}\mathbf{snore}')])(\mathbf{j}')]$

What is peculiar about (2) is that it is still not rendered false in case the extension of $\mathbf{cat}'$ is empty. Rather, it attributes John a contradictory belief due to the empty quantifier $f(\hat{}\mathbf{cat}')$. Such an attribution might still be true assuming John is highly uninformed about logic. (This is for the same reason the classical *de dicto* reading of a sentence like *John believes that some cat is not a cat* is not a contradiction.) I do not know how to evaluate this result.

conceptually important, but it has also empirical implications. For example, from the discussion so far it is not clear what rules out for (101) an analysis like (101a), with the same choice function variable occurring in two different indefinites.

(101) An author saw a teacher.

a. * $\exists f[CH(f) \wedge f(\mathbf{author'})(\lambda x.f(\mathbf{teacher'})(\lambda y.\mathbf{see'}(y)(x)))]$

That (101a) is not a reading of (101) is obvious in a situation where the authors and the teachers are the same set. The proposition in (101a) claims that there is one choice function that picks an author x and a teacher y such that x saw y. But since the two restriction predicates are coreferential x must be equal to y and this amounts to claiming that some author (who is also a teacher) saw herself. This condition is of course stronger than what (101) intuitively requires in this situation. We may conclude that binding of more than one choice function variable by the same existential operator is incorrectly expected by the non-compositional mechanism sketched above. This problem is solved in a compositional setting, as we shall see below.

The question of compositional implementation for choice functions is part of a larger problem: what is the appropriate compositional mechanism for interpreting a given syntactic representation of natural language sentences? A prominent perspective on this problem is the perspective of *Categorial Semantics*. In this framework the question is phrased in the following way:

What are the available rules for composition of types, and what is the semantics of these rules?

A comprehensive attempt to answer this question is of course beyond the scope of the present paper. The reader is referred to van Benthem (1991), Carpenter (1994b), Hendriks (1993), Moortgat (1988) and Nam (1991), among others, for extensive investigations of these topics from different angles. Here, I would like to show that the CF mechanism can be compositionally implemented using three modes of composition that were proposed in addition to standard function application: *Saturation* of a predicate by a quantifier, *Type Lifting* and Restricted *Conditionalization* (= extended Function Composition). A procedure that implements these principles in a restrictive setting is given in appendix B. The following discussion will briefly and informally show the relevance of these operations to the CF mechanism.

5.1 Saturation

The basic problem for compositionality that quantifiers introduce can be exemplified by a simple sentence with a transitive predicate:

(102) Every man likes every woman.

a. [every man] [[every woman] [likes]]

b. $(et)t \;\; [(et)t \;\; e(et)] \;\; \stackrel{?}{\Rightarrow} \;\; t$

Assume that for (102) the syntactic input to semantics is roughly (102a), and that the typing of its elements is the minimal typing in (102b). The question is how to derive a truth value for the sentence. Note that this problem is independent of the problem of scope. Moreover, the same problem appears if there is no rule like "QR" and the structure of (102) is [*every man*] [*likes* [*every woman*]] instead of (102a). One common move in the syntactic literature is to introduce lambda abstractors and free variables into the translation procedure of (102a), or in the syntactic analysis of (102). This results in a representation as in (103).

(103) $[\![\text{every man}]\!]\ (\lambda x.[\![\text{every woman}]\!]\ (\lambda y.[\![\text{likes}]\!](y)(x)))$

This might solve the problem, but takes much of the bite of compositionality: the representation in (103) implicitly assumes a syntactic level of logical representation. For example, the constituent $\lambda y.[\![\text{likes}]\!](y)(x)$ does not have a straightforward modeltheoretic correlate, because of the free variable x. Thus, there is an implicit assumption that in (103) the semantic mechanism does not only compose elementary typed denotations but also uses a syntactic representation in the λ-logical language. Since it is precisely the compositional mechanism for interpreting LF which is the interest of this section, little would be gained here by assuming such an LF' theory.

Many alternatives to this mechanism were proposed in the semantic literature. The one I assume is the idea proposed in Nam (1991) (among others) to define a modeltheoretic operation of *saturation*. The basic definition is the following.

(104) **Saturation**: A quantifier Q of type $(\mathsf{a}t)t$ saturates (the first position) of an $(n+1)$-ary predicate R of type $\mathsf{a}(\mathsf{b}_1...(\mathsf{b}_n t)...)$, $n \geq 0$, by composing the following predicate R' of type $\mathsf{b}_1(...(\mathsf{b}_n t)...)$:

$$R' \stackrel{def}{=} \lambda x_1. \ldots .\lambda x_n.Q(\lambda y.R(y)(x_1)...(x_n))$$

We denote: $Q, R \stackrel{\text{SAT}}{\Rightarrow} R'$

Using saturation, the denotation of (102) is derived as follows:

$$\mathbf{every}'_{(et)((et)t)}(\mathbf{woman}'_{et}) \;\; \mathbf{like}'_{e(et)} \stackrel{\text{SAT}}{\Rightarrow} \lambda x_e.\mathbf{every}'(\mathbf{woman}')(\lambda y_e.\mathbf{like}'(y)(x))$$
$$\mathbf{every}'(\mathbf{man}'_{et}) \; \lambda x.\mathbf{every}'(\mathbf{woman}')(\lambda y.\mathbf{like}'(y)(x)) \stackrel{\text{SAT}}{\Rightarrow}$$
$$\mathbf{every}'(\mathbf{man}')(\lambda x.\mathbf{every}'(\mathbf{woman}')(\lambda y.\mathbf{like}'(y)(x)))$$

Note that saturation always generates a λ-term without free variables, thus the composition is purely modeltheoretic. This mechanism is sufficient to allow composition of quantifiers in their surface order. Nam (1991) is interested also in the interpretation of "QR-ed" structures and therefore uses an indexing mechanism that allows saturation of arbitrary positions of the predicate. For our purposes in this paper, a formalization of full-fledged LF interpretation is not necessary, so this step is not taken.

5.2 CF variables and Type Lifting

In the previous sections it was assumed that an indefinite comes with a free function variable. From a compositional standpoint this is problematic: the names of such variables should be registered and passed on by a long distance mechanism to the site of the existential binding. A more compositional treatment of variables, as proposed for example in Hepple (1991) and Szabolcsi (1987), among others following Quine (1966), views any term φ_τ with a free variable x_σ as a function $\lambda x.\varphi$ of type $\sigma\tau$.

Since an indefinite is basically of type et, the question here is what mechanism turns it into a type $((et)((et)t))((et)t)$: a function from CF's to generalized quantifiers. The mechanism of *type lifting* provides a possible answer. According to this mechanism, we can derive from any type τ the type $(\tau\sigma)\sigma$, for any type σ. Specifically, Partee & Rooth (1983) and many others assume that proper names, although lexically being of type e, can also be interpreted as quantifiers of type $(et)t$. In a similar way, the type $((et)((et)t))((et)t)$ can be derived from et. Semantically, an indefinite with a basic et denotation P can be interpreted as $\lambda f_{(et)((et)t)}.f(P)$. This is hypothetical reasoning of a determiner position (see Partee & Rooth, 1983:p.374). In the present approach this is not a general logical operation but rather one that is limited to indefinites where a lexical determiner is missing. It is therefore expected that this kind of lifting should have a syntactic correlate (e.g. an empty determiner position).

5.3 Variable binding and restricted Conditionalization (extended Function Composition)

Let us abbreviate the choice function type $(et)((et)t)$ as c. Once an indefinite gets the type $c((et)t)$ we would want it to behave like "a potential quantifier", seeking a choice function. For example, in order to obtain an interpretation for (81), we want to allow a type transition as in (105). Once the sentence gets the type ct, this will allow us to apply CF existential closure at the matrix level.

(105) $c((et)t) \;,\; et \;\Rightarrow\; ct$

In fact, (105) is a case of Conditionalization (see van Benthem (1991:ch.4)). We have a situation with two denotations of type $c((et)t)$ and et. Traditional function application does not allow composition here. However, since $(et)t \;\; et \Rightarrow t$ we want composition to behave as if an argument of type c were given to the first function and abstracted over again, in order to derive the type ct. General Conditionalization in this case allows the following derivation:

(106)
$$\dfrac{\dfrac{\dfrac{\mathsf{ab} \quad [\mathsf{a}]^i}{\mathsf{b}} \quad \mathsf{c}}{\mathsf{d}}}{\mathsf{ad}}\ \text{COND, withdraw premise } i$$

(105) is derived by substituting: $\mathsf{a} = c$, $\mathsf{b} = (et)t$, $\mathsf{c} = et$ and $\mathsf{d} = t$. However, since in general a conditionalization rule as used in (106) is too strong (see Hendriks (1993:68-70)) we restrict it by admitting only a "range change" rule RCOND (Restricted Conditionalization) as in (107) (in sequent format), which is a less general rule than the COND full introduction rule.

(107)
$$\text{RCOND}\ \dfrac{\mathsf{b} \;,\; \mathsf{c} \;\Rightarrow\; \mathsf{d}}{\mathsf{ab} \;,\; \mathsf{c} \;\Rightarrow\; \mathsf{ad}}$$

Note that RCOND extends classical Function Composition: if we substitute bd for c, then (107) reduces to $\mathsf{ab} \;,\; \mathsf{bd} \;\Rightarrow\; \mathsf{ad}$.

The semantics for (107) is as follows:

(108)
$$\text{RCOND}\ \dfrac{y \;,\; X \;\Rightarrow\; \psi}{\varphi \;,\; X \;\Rightarrow\; \lambda x.(\lambda y.\psi)(\varphi(x))}$$

RCOND allows the type transition in (105) with the following (appropriate) semantics:
$\lambda f.f(\mathbf{woman}') \;,\; \mathbf{smile}' \;\Rightarrow\; \lambda f.f(\mathbf{woman}')(\mathbf{smile}')$
To see that, consider the following inference:

$$\dfrac{Q_{(et)t} \;,\; \mathbf{smile}'_{et} \;\Rightarrow\; (Q(\mathbf{smile}'))_t}{(\lambda f_c.f(\mathbf{woman}'))_{c((et)t)} \;,\; \mathbf{smile}'_{et} \;\Rightarrow\; (\lambda g_c.(\lambda Q.Q(\mathbf{smile}'))((\lambda f.(f(\mathbf{woman}')))(g)))_{ct}}\ RCOND$$

and note that the resulting term is equivalent to $\lambda f.f(\mathbf{woman}')(\mathbf{smile}')$.

Generally, recursive application of RCOND allows the following derivation:
$\mathsf{a}_1(\ldots(\mathsf{a}_n\mathsf{b})\ldots) \;,\; \mathsf{c} \;\Rightarrow\; \mathsf{a}_1(\ldots(\mathsf{a}_n\mathsf{d})\ldots)$, provided that $\mathsf{b} \;,\; \mathsf{c} \;\Rightarrow\; \mathsf{d}$.
Thus, an arbitrary number of CF variables can be "passed on" by the compositional mechanism. However, for many cases even RCOND is a too liberal rule of composition. Some basic problems for the LP calculus of van Benthem (1991) that were pointed out by Hendriks appear also with RCOND instead of the COND rule of LP. To avoid that, two different functional type constructors are used in appendix B. For the sake of the general discussion I ignore this point here.

5.4 Existential Choice Closure

We saw what assumptions are required for representing the simple sentence (105) as a predicate over choice functions. According to (108), (105) gets the following meaning:

(109) $\lambda f_c.f(\mathbf{woman'})(\mathbf{smile'})$

To obtain a truth value, we employ an operation of existential closure for choice functions. This is application of a generalized quantifier over CF's. Thus, if $\mathbf{E}_{(ct)((ct)t)}$ is the standard existential determiner over the domain D_c as defined in (110), [44] then $\mathbf{E}(CH_{ct})$ is the required quantifier, as given in (111).

(110) $\mathbf{E}_{(ct)((ct)t)} \stackrel{def}{=} \lambda A_{ct}.\lambda B_{ct}.\exists f_c[A(f) \wedge B(f)]$

(111) $\mathbf{E}(CH) = \lambda P_{ct}.\exists f_c[CH(f) \wedge P(f)]$

Applying $\mathbf{E}(CH)$ to (109) we get:

$$\mathbf{E}(CH)(\lambda f_c.f(\mathbf{woman'})(\mathbf{smile'})) \Leftrightarrow \exists f[CH(f) \wedge f(\mathbf{woman'})(\mathbf{smile'})]$$

which is the intended meaning. The application of $\mathbf{E}(CH)$ is assumed to take place in the translation procedure (see appendix B).

5.5 An illustrative analysis

An illustration of a compositional analysis of a simplified LF for sentence (89) is given in figure 1.

The tree interpretation in figure 1 is a basic analysis, without any application of $\mathbf{E}(CH)$. Consequently, it underspecifies the scope of the indefinite and is not of type t. Application of $\mathbf{E}(CH)$ to the denotations of nodes 1 and 2 leads to a wide scope reading. Application to other nodes leads to a narrow scope reading. For example:

(112) $\mathbf{E}(CH)$ at node 2:

$\mathbf{E}(CH)$, $\lambda f.\lambda B_{et}.\forall x[(\mathbf{artist'}(x) \wedge f(\mathbf{city'})(\lambda y.\mathbf{born\_in'}(y)(x))) \rightarrow B(x)] \stackrel{\text{SAT}}{\Rightarrow}$
$\lambda B.\mathbf{E}(CH)(\lambda f.\forall x[(\mathbf{artist'}(x) \wedge f(\mathbf{city'})(\lambda y.\mathbf{born\_in'}(y)(x))) \rightarrow B(x)]) =$
$\lambda B.\exists f[CH(f) \wedge \forall x[(\mathbf{artist'}(x) \wedge f(\mathbf{city'})(\lambda y.\mathbf{born\_in'}(y)(x))) \rightarrow B(x)]]$

Application to $\mathbf{famous'}$:

$\exists f[CH(f) \wedge \forall x[(\mathbf{artist'}(x) \wedge f(\mathbf{city'})(\lambda y.\mathbf{born\_in'}(y)(x))) \rightarrow \mathbf{famous'}(x)]]$

(113) $\mathbf{E}(CH)$ at node 3:

$\mathbf{E}(CH)$, $\lambda f.f(\mathbf{city'}) \stackrel{\text{SAT}}{\Rightarrow}$
$\lambda A.\exists f[CH(f) \wedge f(\mathbf{city'})(A)]$

The rest of the derivation is standard, using saturation and application, and leads to:

$\forall x[[\mathbf{artist'}(x) \wedge \exists f(CH(f) \wedge f(\mathbf{city'})(\lambda y.\mathbf{born\_in'}(y)(x)))] \rightarrow \mathbf{famous'}(x)]$

Now it is clear why problems as illustrated above in (101) do not occur in a compositional semantics: the existential closure operator is an unary quantifier, which can saturate only one argument position of a predicate over choice functions.

The above discussion is too sketchy in the description of the restricted conditionalization rule and in the relations between syntax and semantics (the translation procedure). One system that implements the

[44] Since c is the $(et)((et)t)$ type for determiners over the D_e domain, $\mathbf{E}$ is actually a determiner over determiners.

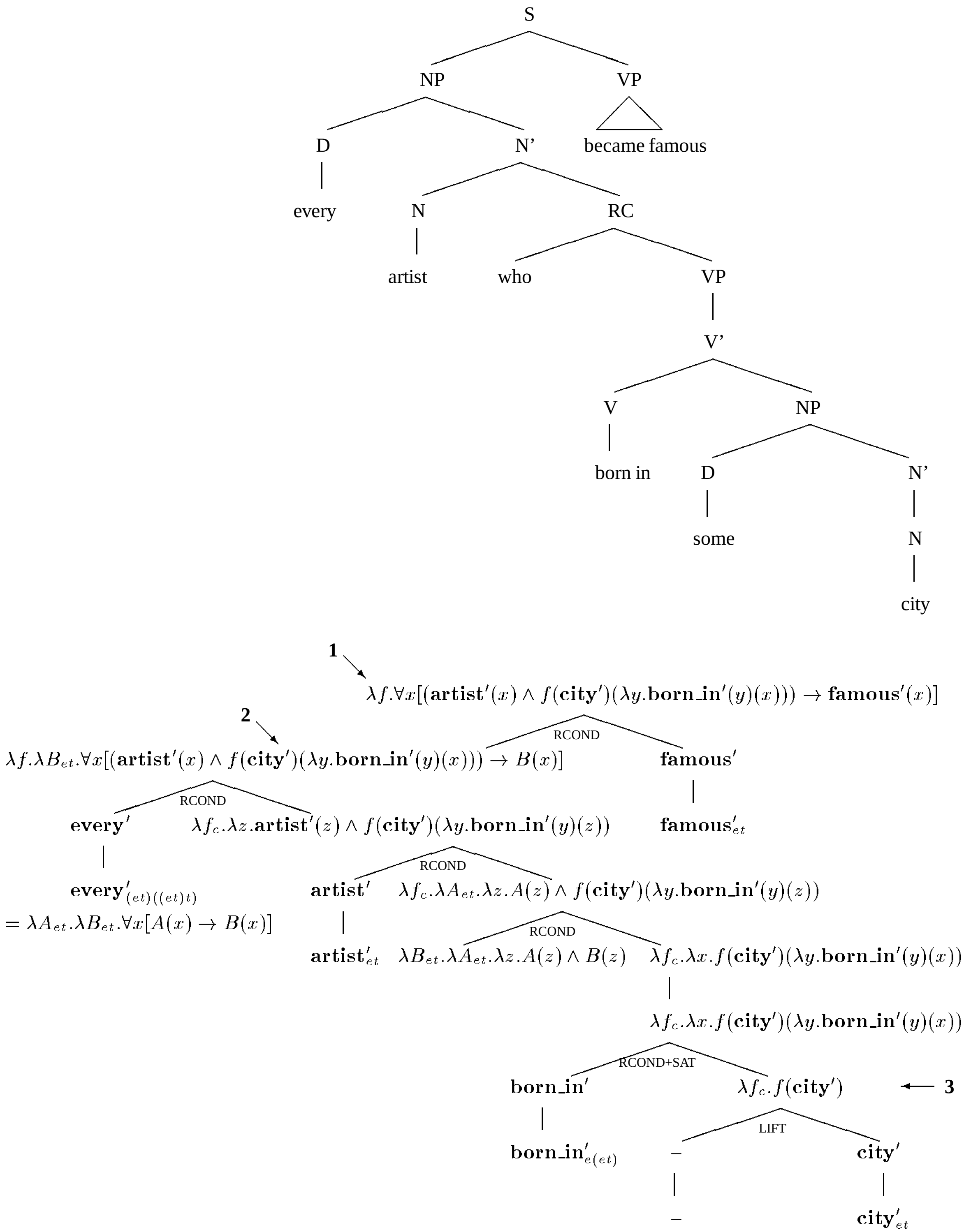


Figure 1: An illustration of LF interpretation

proposed principles is given in appendix B. Of course, the design of a general compositional mechanism for semantic interpretation remains a huge logical and linguistic problem. The points made above and the system in appendix B serve two objectives of the present enterprise: (i) showing that a compositional mechanism for interpretation using choice functions is within reach, and (ii) pointing to the intimate relation between the CF approach and the standard existential quantification of GQ theory.

It is the second general point with which this paper will be concluded.

6 Epilogue: back to generalized quantifiers

The proposal to treat indefinites using choice functions might seem to be making a sharp distinction between quantificational theories of indefinites (*à la* Russell) and theories that treat indefinites as individuals (*à la* Hilbert). In developing a theory of the second kind, one might think that we left the fruitful framework of generalized quantifiers and moved to an unknown territory. Some indications that this is actually not the case, however, can be found in sections 4 and 5. In the proposed CF mechanism indefinites are still treated as GQ's, but they are of a special kind in that they contain a "free CF variable". What does this mean? In a compositional setting the picture becomes clearer: getting back to figure 1, we saw that the system allows application of $\mathbf{E}(CH)$ already at the indefinite NP level, i.e. in the node marked 3. Observing that, it is expected that the NP denotation, created without any free variables, should reduce to a standard existential GQ. This is indeed the case:

Proposition 6.1 *For every predicate* A_{et}*:* $\mathbf{E}(CH)\ ,\ \lambda f_c.f(A) \stackrel{\text{SAT}}{\Rightarrow} \lambda P_{et}.\exists x[P(x) \wedge A(x)]$

Proof*: We saw already that* $\mathbf{E}(CH)\ ,\ \lambda f_c.f(A) \stackrel{\text{SAT}}{\Rightarrow} \lambda P.\exists f[CH(f) \wedge f(A)(P)]$. *And we saw that by the choice definition (87)* $\exists f[CH(f) \wedge f(A)(P)] \Leftrightarrow \exists x[P(x) \wedge A(x)]$.

Turning to plurals, note that $\mathbf{E}(CH)$ can apply to a non-distributed or to a distributed NP denotation. These two options lead to the following results for the NP *three women*:

Corollary 6.2 $\mathbf{E}(CH)\ ,\ \lambda f.f(\mathbf{three}'(\mathbf{women}')) \stackrel{\text{SAT}}{\Rightarrow} \lambda P_{et}.\exists x[P(x) \wedge \mathbf{three}'(\mathbf{women}')(x)]$
$\stackrel{def}{=} \mathbf{three}^{c}_{(et)((et)t)}(\mathbf{women}'_{et})$

Proposition 6.3 $\mathbf{E}(CH)\ ,\ D\ ,\ \lambda f.f(\mathbf{three}'(\mathbf{women}')) \stackrel{\text{SAT+RCOND}}{\Rightarrow} \lambda P_{et}.|P \cap \mathbf{woman}'| \geq 3$
$\stackrel{def}{=} \mathbf{three}^{d}_{(et)((et)t)}(\mathbf{woman}'_{et})$

Proof*: See appendix A.*

Now, $\mathbf{three}^d$ is the distributive determiner denotation for *three* from Barwise & Cooper (1981) (equivalent to *at least three*). [45] $\mathbf{three}^c$ becomes the first collective determiner reading of *three* in Scha (1981) (here without the explicit higher set-theoretical typing). Thus, propositions 6.1–6.3 locate the CF mechanism within the GQ framework in the following sense:

1. Simple indefinite NP's are basically "slashed" GQ's , waiting for a determiner value.
2. The existential import of simple indefinites is "set free" in the CF mechanism by the DRT like separation between $\mathbf{E}(CH)$ and the NP denotation.

[45] There is a disagreement between Barwise & Cooper and Scha on the question whether a distributive numeral *n* is equivalent to *at least n* (B&C) or to *exactly n* (Scha). As noted above, I take the B&C/Grice choice to be preferable. In fact, a treatment of numerals as modifiers plus existential quantification, as argued for in this paper, must assume the B&C monotone semantics of simple numerals.

3. When $\mathbf{E}(CH)$ is "glued back" to the NP denotation, it leads to standard GQ's in distributive/collective systems.

7 Conclusions

The investigation of the scopal semantics of indefinites is a worthwhile enterprise for both syntactic and semantic theories. Syntactic theories can benefit from a better semantic understanding of indefinites: a mechanism like the one developed in this paper shows that there is no need for unconstrained structural operations in order to account for the free existential scope of indefinites. On the other hand, semantic theories, as always, gain significant restrictiveness from structural considerations in a compositional framework.

The purpose of the present study was to show some advantages of such a project. Further research is certainly needed in order to evaluate the conclusions reached and to explore their implications. I hope that the first steps for progress in this respect have been made.

Acknowledgements
This paper develops the preliminary work in Winter (1995b). Both papers would not have been written if not for many extensive discussions with Tanya Reinhart, to whom I am deeply grateful. Herman Hendriks provided invaluable remarks on almost every word of a previous draft, which helped very much in improving it. Special thanks also to Johan van Benthem, Jaap van der Does, Danny Fox, Nissim Francez, Klaus von Heusinger, Shalom Lappin and Eddy Ruys for substantial critique on various points. Thanks to Misha Becker, Filippo Beghelli, Dorit Ben-Shalom, Gennaro Chierchia, Donka Farkas, Jeroen Groenendijk, Irene Heim, Ed Keenan, Ruth Kempson, Wilfried Meyer Viol, Bill Philip, Barry Schein, Tim Stowell, Anna Szabolcsi and Henk Verkuyl for interesting discussions. Finally, I am indebted to Martin Stokhof and two anonymous referees of *Linguistics and Philosophy* for providing helpful and penetrating remarks.

All remaining errors and misconceptions are of course mine.

Appendices

A Proofs of propositions

Proof 4.1: Assume that D is a conservative determiner and that the following proposition holds:
$\exists CH_{((et)e)t}\forall A_{et}\forall B_{et}[[\exists f[CH(f)\wedge A(f(B))]] \leftrightarrow D(B)(A)]$ (1)
Assume CH_0 is a constant satisfying (1), such that the following holds:
$\forall A\forall B[[\exists f[CH_0(f)\wedge A(f(B))]] \leftrightarrow D(B)(A)]$ (2)
Instantiate in (2) $A = B = \emptyset_{et} \stackrel{def}{=} \lambda x_e.\bot$:
$[\exists f[CH_0(f)\wedge \emptyset(f(\emptyset))]] \leftrightarrow D(\emptyset)(\emptyset)$
or: $\bot \leftrightarrow D(\emptyset)(\emptyset)$
By conservativity of D: $\forall A[D(\emptyset)(A)\leftrightarrow D(\emptyset)(\emptyset)]$
That is $\forall A[D(\emptyset)(A)\leftrightarrow \bot]$ (3)
Instantiate in (2) $A = \mathbf{U}_{et} \stackrel{def}{=} \lambda x_e.\top$, $B=\emptyset$:
$[\exists f[CH_0(f)\wedge \mathbf{U}(f(\emptyset))]] \leftrightarrow D(\emptyset)(\mathbf{U})$
By (3): $D(\emptyset)(\mathbf{U})\leftrightarrow \bot$
We get: $[\neg\exists f CH_0(f)]\vee \mathbf{U}=\emptyset$
Therefore: $\forall A\forall B[[\exists f[CH_0(f)\wedge A(f(B))]] \leftrightarrow \bot]$

And by (2): $\forall A \forall B[\neg D(B)(A)]$
Conclusion: $[\exists CH_{((et)e)t}\ \forall A_{et}\ \forall B_{et}\ [\ \exists f_{(et)e}[CH(f) \wedge A(f(B))] \leftrightarrow\ D(B)(A)\ ]\ ] \rightarrow \forall A\ \forall B\ [\neg D(B)(A)]$

Proof 6.3: $D\ ,\ \ \lambda f.f(\mathbf{three}'(\mathbf{women}'))\ \stackrel{\text{RCOND}}{\Rightarrow}\ \lambda f.D(f(\mathbf{three}'(\mathbf{women}')))$
$\mathbf{E}(CH)\ ,\ \ \lambda f.D(f(\mathbf{three}'(\mathbf{women}')))\ \stackrel{\text{SAT}}{\Rightarrow}\ \lambda P_{et}.\exists f[CH(f) \wedge D(f(\mathbf{three}'(\mathbf{women}')))(P)]$
By definition in footnote 41:
$= \ \lambda P.\exists f[CH(f) \wedge \exists x[[\text{NAG}(f(\mathbf{three}'(\mathbf{women}')))(x) \wedge D'(x)(P)] \vee$
$[[\neg \exists x \text{NAG}(f(\mathbf{three}'(\mathbf{women}')))(x)] \wedge f(\mathbf{three}'(\mathbf{women}'))(P)]]]$ (1)
By definition $\mathbf{three}'(\mathbf{women}') \cap \mathcal{A} = \emptyset$. Assume $\mathbf{three}'(\mathbf{women}') \neq \emptyset$. Then by definition of CH:
(1) $= \ \lambda P.\exists x[\mathbf{three}'(\mathbf{women}')(x) \wedge D'(x)(P)]$
and by definition of $\mathbf{three}'$, $\mathbf{women}'$ and D':
$= \ \lambda P.|\mathbf{woman}' \cap P| \geq 3$
Assume now $\mathbf{three}'(\mathbf{women}') = \emptyset$, that is $|\mathbf{woman}'| < 3$. We get:
(1) $= \ \lambda P.\bot \ = \ \lambda P.|\mathbf{woman}' \cap P| \geq 3$

B A procedure for compositional interpretation using choice functions

B.1 Overview

The system defined below is based on a λ-theoretical language used to describe modeltheoretic objects. This language is a standard extensionally typed λ-calculus except for the fact that it includes *two functional type constructors*: ”,” and ”$\rightarrow$”, with identical semantics. Composition of terms is done using a fragment of van Benthem (1991)’s LP calculus. In addition to the rules of *application* and *permutation*, there is a rule of *saturation* (n-ary predicate – quantifier composition) and a rule of *restricted conditionalization* for the constructor ”$\rightarrow$” (both rules are special cases of van Benthem’s Conditionalization). The latter rule takes care of “variable percolation” and thus only choice functions are introduced with the ”$\rightarrow$” constructor (see remark below). The input to the system is an *LF tree*: a simple binary tree with lexical translations of the leaves. It is assumed that introduction of choice functions is lexically encoded in the denotation of an empty category ϕ_L with the semantics of *type lifting*. Distributivity is also syntactically introduced by an empty category – ϕ_D, which is optionally stipulated in the syntax of the NP. *Translation* is bottom-up in the LF tree, composing semantic values using the type calculus into a denotation of the dominating node. This process embodies a procedure of *existential choice closure* that optionally composes with any derived semantic value.

Remark: The functional construct $(\mathsf{a} \rightarrow \mathsf{b})$ must be distinguished from the weaker construction (a, b) which is given to lexical items. Problems similar to those revealed in Hendriks (1993:69-70) for the LP calculus would reappear if $\rightarrow$ were given to lexical items.
For example, type $(e \rightarrow t) \rightarrow ((e \rightarrow t) \rightarrow t)$ for *every* would result in an absurd derivation:
$\lambda A.\lambda B.A \subseteq B \quad P \ \stackrel{\text{RCOND}}{\Rightarrow}\ \lambda A.A \subseteq P$
In the system below lexical items should be given only types with the weaker constructor (a, b), while the $\rightarrow$ constructor is reserved to choice functions introduced by the empty node ϕ_L. This prevents undesired application of RCOND where choice functions are not involved.

B.2 Definitions

Types:

1. e and t are types.

2. If τ and σ are types then (τ, σ) and $(\tau \rightarrow \sigma)$ are types.

Terms: For every type τ, let $VAR(\tau)$ be an infinite set of variables, $CONS(\tau)$ a set of constants. φ is a term of type τ (abbr. φ_τ) iff:

$\varphi \in VAR(\tau)$ or $\varphi \in CONS(\tau)$,

φ is of the form $\psi_{\sigma,\tau}(\chi_\sigma)$ or φ is of the form $\psi_{\sigma\rightarrow\tau}(\chi_\sigma)$ (application),

φ is of the form $\lambda x_{\sigma_1}.\psi_{\sigma_2}$, where $x \in VAR(\sigma_1)$ and ($\tau = (\sigma_1, \sigma_2)$ or $\tau = (\sigma_1 \rightarrow \sigma_2)$) (abstraction),

or φ is of the form $(\psi_\sigma = \chi_\sigma)$ and $\tau = t$ (identity).

First-order logical constants $\top,\ \bot,\ \neg,\ \wedge,\ \vee,\ \rightarrow,\ \forall,\ \exists$ can be defined in this language (see e.g. van Benthem (1991:p.7)). We will assume the standard semantics. The type constructors "," and "$\rightarrow$" both have the standard semantics of the functional constructor. We switch freely between terms that are semantically equivalent, indicating equivalence using identity "=" in the meta-language.

Type transitions + semantics (natural deduction format):

APP $\dfrac{(\mathsf{a}\ op\ \mathsf{b}) \quad \mathsf{a}}{\mathsf{b}}$ $op \in \{,\ \rightarrow\}$ $\qquad \dfrac{x \quad y}{x(y)}$

SAT $\dfrac{(\mathsf{a}, t), t \quad \mathsf{a}\ op_1(\mathsf{b}_1\ op_2(\mathsf{b}_2\ ...\ op_n(\mathsf{b}_n\ op_{n+1}\ t)\ ...))}{\mathsf{b}_1\ op_2(\mathsf{b}_2\ ...\ op_n(\mathsf{b}_n\ op_{n+1}\ t)\ ...\ )}$ $n \geq 0$, and for all i s.t. $1 \leq i \leq n+1$: $op_i \in \{,\ \rightarrow\}$

$$\dfrac{Q \qquad\qquad R}{\lambda x_1.\lambda x_2.\ ...\ .\lambda x_n.Q(\lambda y.R(y)(x_1)(x_2)...(x_n))}$$

RCOND^i $\dfrac{\mathsf{a} \rightarrow \mathsf{b} \quad \begin{matrix}[\mathsf{b}]^i \ \ ... \\ \vdots \\ \mathsf{c}\end{matrix}}{\mathsf{a} \rightarrow \mathsf{c}}$ withdraw premise i $\qquad \dfrac{\varphi \quad \begin{matrix}[y]^i \ \ ... \\ \vdots \\ \psi\end{matrix}}{\lambda x_{\mathsf{a}}.(\lambda y.\psi)(\varphi(x))}$

PERM $\dfrac{\mathsf{a} \quad \mathsf{b}}{\mathsf{b} \quad \mathsf{a}}$ $\qquad \dfrac{x \quad y}{y \quad x}$

Derivation: We denote $\mathsf{a}\ \mathsf{b} \vdash \mathsf{c}$, and $\varphi\ \ \psi \Rightarrow \chi$ iff type c with semantics χ is derivable from types a and b with semantics φ and ψ using the rules above.

Plurality constants: $\mathcal{A}_{e,t}\ \Pi_{e,(e,t)}$. Read $\mathcal{A}(x)$ as "x is an atom" and $\Pi(x)(y)$ as "x is an atomic part of y". We assume that (at least) the following holds:

(i) $\exists x \mathcal{A}(x)$ and

(ii) $\forall x \forall y [\Pi(x)(y) \rightarrow \mathcal{A}(x)]$.

Further, we define the relation of *Non-Atomic Generator* (NAG) between quantifiers and individuals:

$\mathrm{NAG}_{((e,t),t),(e,t)} \stackrel{def}{=} \lambda Q.\lambda x.\neg\mathcal{A}(x) \wedge Q = \lambda P.P(x)$

And finally, we define *the distributor* of a quantifier as follows:

$D_{((e,t),t),((e,t),t)} \stackrel{def}{=} \lambda Q.\lambda P.\exists x[\mathrm{NAG}(Q)(x) \wedge \forall y[\Pi(y)(x) \rightarrow P(y)] \vee [[\neg\exists x \mathrm{NAG}(Q)(x)] \wedge Q(P)]$

LF Tree: A 3-tuple $\langle \mathsf{T}, \mathsf{cat}, tr_{\mathsf{lex}} \rangle$ is called an *LF tree* iff:

1. T is a binary tree, i.e. it is a tuple $\langle V, daughter \rangle$, where V is the set of vertices and $daughter$ is a function $V \rightarrow P(V)$ with the appropriate properties (the order of daughters is immaterial for our purposes).

2. cat is a function from the internal nodes of T to the set of syntactic categories.

3. tr_{lex} is a function from the leaves of T to the set $\{\langle \mathsf{a}, \varphi \rangle | \varphi$ is a term of type $\mathsf{a}\}$.

Existential Choice Closure: Let us abbreviate the type $((e,t),((e,t),t))$ as c, and the term $\lambda X_\tau.\bot$ as $\emptyset_{\tau,t}$.

$$CH_{c,t} \stackrel{def}{=} \lambda f_c.\forall P_{e,t}[P \neq \emptyset \rightarrow \exists x_e(P(x) \wedge f(P) = \lambda A_{e,t}.A(x))] \wedge f(\emptyset_{e,t}) = \emptyset_{(e,t),t}$$

$$\mathbf{E}_{(c,t),((c,t),t)} \stackrel{def}{=} \lambda A_{c,t}.\lambda B_{c,t}.\exists f_c[A(f) \wedge B(f)]$$

Let TR be a set of tuples of the form $\langle \mathsf{a}, \varphi \rangle$, where φ is a term of type a. The *existential choice closure* of TR is the set $CLOS(TR) \supseteq TR$, recursively defined as follows:

$$clos_0(TR) \stackrel{def}{=} TR$$

$$clos_{n+1}(TR) \stackrel{def}{=} \{\langle \mathsf{a}, \varphi \rangle \mid \text{for some } \langle \mathsf{a}', \varphi' \rangle \in clos_n(TR) : \mathbf{E}(CH) \quad \varphi'_{\mathsf{a}'} \Rightarrow \varphi_{\mathsf{a}}\}$$

$$CLOS(TR) \stackrel{def}{=} \bigcup_{n=0}^{\infty} clos_n(TR)$$

Translation: Let $\langle \mathsf{T}, \mathsf{cat}, tr_{\mathsf{lex}} \rangle$ be an LF tree. The *basic translation set* $TR_B(v)$ and the *translation set* $TR(v)$ of a node v in T are recursively defined as follows:

1. in case *daughter*$(v) = \emptyset$: $\quad TR_B(v) \stackrel{def}{=} \{tr_{\mathsf{lex}}(v)\}$

2. in case *daughter*$(v) = \{v_1\}$: $\quad TR_B(v) \stackrel{def}{=} TR(v_1)$

3. in case *daughter*$(v) = \{v_1, v_2\}$:

   $TR_B(v) \stackrel{def}{=} \{\langle \mathsf{c}, \chi \rangle \mid \exists \varphi_{\mathsf{a}} \exists \psi_{\mathsf{b}}[\langle \mathsf{a}, \varphi \rangle \in TR(v_1) \wedge \langle \mathsf{b}, \psi \rangle \in TR(v_2) \wedge \varphi_{\mathsf{a}}\ \psi_{\mathsf{b}} \Rightarrow \chi_{\mathsf{c}}]\}$

The translation set of v, $TR(v) \overset{def}{=} CLOS(TR_B(v))$, the existential choice closure of the basic translation of v.

Let v_0 be the root node of T. Then the *complete translation set* of T is $\{\langle t, \varphi\rangle \mid \langle t, \varphi\rangle \in TR(v_0)\}$, all the terms in the translation of v_0 that denote truth values.

B.3 Example

(100) If three workers in our staff have a baby soon we will have to face some hard organizational problems.

We stipulate the LF tree and the lexical translations below. For convenience the analysis of some irrelevant nodes is spared and they are condensed as lexical items.

$[_{\mathrm{S}}\ [_{\mathrm{S}'_1}$ if $[_{\mathrm{S}_1}\ [_{\mathrm{NP}_1}\ \phi_D\ [\ \phi_L\ [_{\mathrm{N}'}$ three workers]]] $[_{\mathrm{VP}}$ have $[_{\mathrm{NP}_2}\ \phi_L$ baby]]]] $[_{\mathrm{S}_2}$ problems]]

$tr_{\mathsf{lex}}(\mathrm{if}) = \langle\, t, (t,t)\ ,\ \mathbf{if'}\,\rangle$
$\mathbf{if'} \overset{def}{=} \lambda\varphi.\lambda\psi.\varphi \rightarrow \psi$

$tr_{\mathsf{lex}}(\phi_D) = \langle\ ((e,t),t), ((e,t),t)\ ,\ D\,\rangle$

$tr_{\mathsf{lex}}(\phi_L) = \langle\ (e,t), (c \rightarrow ((e,t),t))\ ,\ L\,\rangle$
$L \overset{def}{=} \lambda P_{e,t}.\lambda f_c.f(P)$

$tr_{\mathsf{lex}}(\mathrm{three}) = \langle\ (e,t),(e,t)\ ,\ \mathbf{three'}\,\rangle$
$\mathbf{three'} \overset{def}{=} \lambda P_{e,t}.\lambda x_e.P(x) \wedge \exists 3! z\ \Pi(z)(x)$

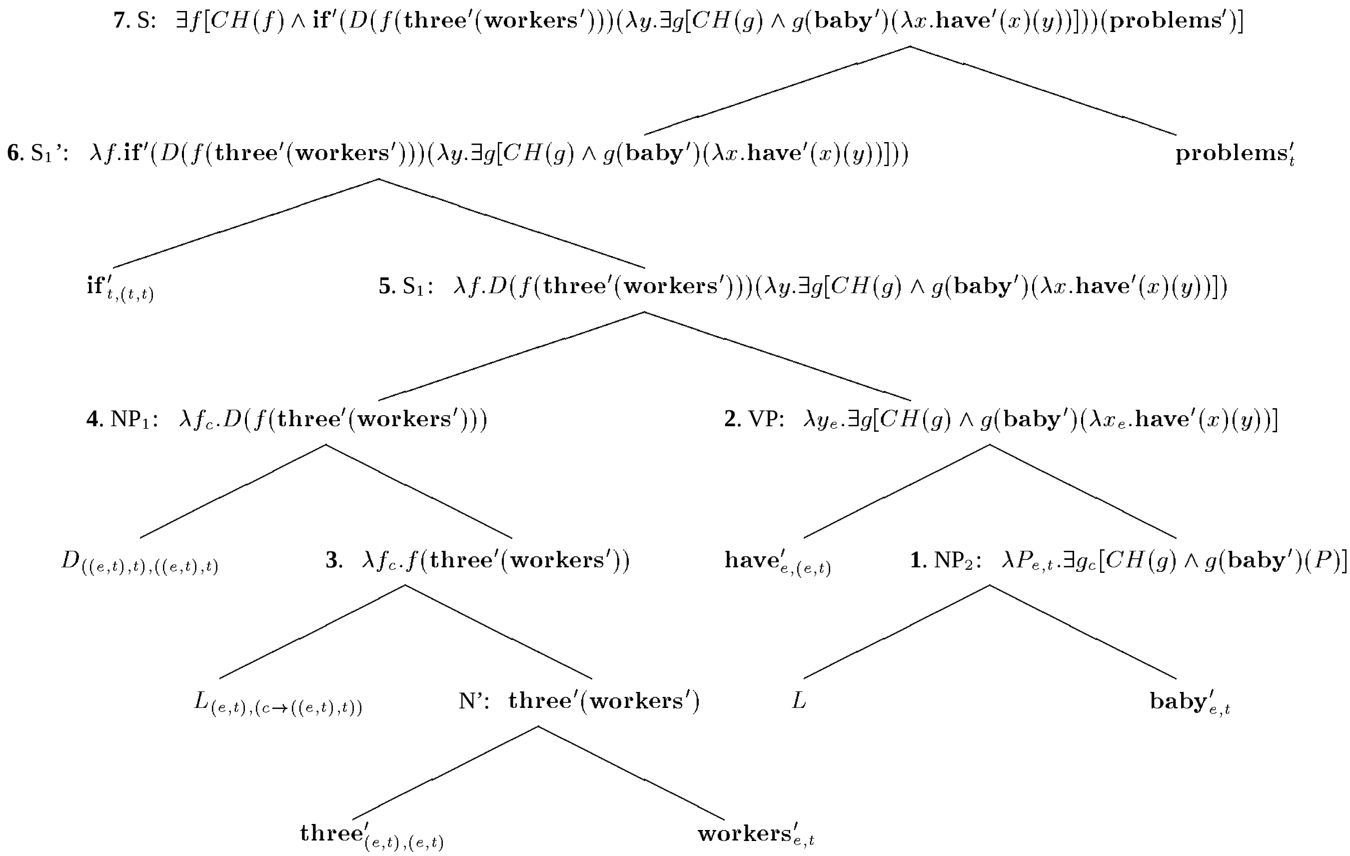
$tr_{\mathsf{lex}}(\mathrm{workers}) = \langle\ (e,t)\ ,\ \mathbf{workers'}\,\rangle$
$\mathbf{workers'} \overset{def}{=} \lambda x_e.\forall y_e[\Pi(y)(x) \rightarrow \mathbf{worker'}_{e,t}(y)]\ \wedge\ \exists 2z\ \Pi(z)(x)$

$tr_{\mathsf{lex}}(\mathrm{have}) = \langle\ (e^2, (e^1, t))\ ,\ \mathbf{have'}\,\rangle$

$tr_{\mathsf{lex}}(\mathrm{baby}) = \langle\ (e,t)\ ,\ \mathbf{baby'}\,\rangle$

$tr_{\mathsf{lex}}(\mathrm{problems}) = \langle\ t\ ,\ \mathbf{problems'}\,\rangle$

Let us observe that the following proposition is among the tree translations of this structure.
$\exists f[CH(f) \wedge \mathbf{if'}(D(f(\mathbf{three'}(\mathbf{workers'}))))(\lambda y.\exists g[CH(g) \wedge g(\mathbf{baby'})(\lambda x.\mathbf{have'}(x)(y))]]))(\mathbf{problems'})]$
To see that, the following labeled tree contains the relevant node translations and the numbers in the internal nodes point to the corresponding inference below responsible for the derivation of the translation.



Remark: in the derivations 2,4,6 and 7 below there are some necessary PERM applications that are omitted to increase readability.

1.

$$\frac{(\mathbf{E}(CH))_{(c,t),t} \qquad \dfrac{L_{(e,t),(c\to((e,t),t))} \quad \mathbf{baby}'_{e,t}}{(L(\mathbf{baby}'))_{c\to((e,t),t)}}\ \text{APP (i)}}{(\lambda P_{e,t}.\mathbf{E}(CH)(\lambda g_c.L(\mathbf{baby}')(g)(P)))_{(e,t),t}}\ \text{SAT (ii)}$$

by definition of $\mathbf{E}(CH)$ and L:

$= \lambda P.\exists g[CH(g) \wedge g(\mathbf{baby}')(P)]$

Step (i) in the derivation shows that $L(\mathbf{baby}') \in TR_B(\text{NP}_2)$

Step (ii) shows that $\lambda P.\exists g[CH(g) \wedge g(\mathbf{baby}')(P)] \in clos_1(TR_B(\text{NP}_2)) \subseteq CLOS(TR_B(\text{NP}_2)) =$

$TR(\mathrm{NP}_2)$

2.

$$\frac{\mathbf{have}'_{e,(e,t)} \quad (\lambda P.\exists g[CH(g) \wedge g(\mathbf{baby}')(P)])_{(e,t),t}}{(\lambda y_e.(\lambda P.\exists g[CH(g) \wedge g(\mathbf{baby}')(P)])(\lambda x_e.\mathbf{have}'(x)(y)))_{e,t}}\ \text{SAT}$$

$$= \lambda y.\exists g[CH(g) \wedge g(\mathbf{baby}')(\lambda x.\mathbf{have}'(x)(y))]$$

3.

$L(\mathbf{three}'(\mathbf{workers}')) \ = \ \lambda f_c.f(\mathbf{three}'(\mathbf{workers}'))$ by definition of L.

4.

$$\frac{\dfrac{D_{((e,t),t),((e,t),t)} \quad [Q_{(e,t),t}]^1}{(D(Q))_{(e,t),t}}\ \text{APP} \qquad (\lambda f_c.f(\mathbf{three}'(\mathbf{workers}')))_{c\to((e,t),t)}}{(\lambda h_c.(\lambda Q.D(Q))((\lambda f.f(\mathbf{three}'(\mathbf{workers}')))(h)))_{c\to((e,t),t)}}\ \text{RCOND}^1$$

simplification and renaming:

$$= \ \lambda f.D(f(\mathbf{three}'(\mathbf{workers}')))$$

5.

$$\frac{(\lambda f_c.D(f(\mathbf{three}'(\mathbf{workers}))))_{c\to((e,t),t)} \qquad \dfrac{[Q_{(e,t),t}]^1 \quad (\lambda y.\exists g[CH(g) \wedge g(\mathbf{baby}')(\lambda x.\mathbf{have}'(x)(y))])_{e,t}}{(Q(\lambda y.\exists g[CH(g) \wedge g(\mathbf{baby}')(\lambda x.\mathbf{have}'(x)(y))]))_t}\ \text{APP}}{(\lambda h_c.(\lambda Q.Q(\lambda y.\exists g[CH(g) \wedge g(\mathbf{baby}')(\lambda x.\mathbf{have}'(x)(y))]))(\lambda f.D(f(\mathbf{three}'(\mathbf{workers})))(h)))_{c\to t}}\ \text{RCOND}^1$$

simplification and renaming:

$$= \ \lambda f.D(f(\mathbf{three}'(\mathbf{workers}')))(\lambda y.\exists g[CH(g) \wedge g(\mathbf{baby}')(\lambda x.\mathbf{have}'(x)(y))])$$

6.

$$\frac{\dfrac{\mathbf{if}'_{t,(t,t)} \quad [\varphi_t]^1}{(\mathbf{if}'(\varphi))_{t,t}}\ \text{APP} \qquad (\lambda f_c.D(f(\mathbf{three}'(\mathbf{workers}')))(\lambda y.\exists g[CH(g) \wedge g(\mathbf{baby}')(\lambda x.\mathbf{have}'(x)(y))]))_{c\to t}}{(\lambda h_c.(\lambda \varphi_t.(\mathbf{if}'(\varphi)))((\lambda f.D(f(\mathbf{three}'(\mathbf{workers}')))(\lambda y.\exists g[CH(g) \wedge g(\mathbf{baby}')(\lambda x.\mathbf{have}'(x)(y))]))(h)))_{c\to(t,t)}}\ \text{RCOND}^1$$

simplification and renaming:

$$= \lambda f.\mathbf{if}'(D(f(\mathbf{three}'(\mathbf{workers}')))(\lambda y.\exists g[CH(g) \wedge g(\mathbf{baby}')(\lambda x.\mathbf{have}'(x)(y))]))$$

7.

$$\dfrac{(\mathbf{E}(CH))_{(c,t),t} \quad \dfrac{(\lambda f_c.\mathbf{if}'(D(f(\mathbf{three}'(\mathbf{workers}')))(\lambda y.\exists g[CH(g) \wedge g(\mathbf{baby}')(\lambda x.\mathbf{have}'(x)(y))])))_{c\to(t,t)} \quad \dfrac{[O_{t,t}]^1 \quad \mathbf{problems}'_t}{(O(\mathbf{problems}'))_t}\text{APP}}{(\lambda h_c.(\lambda O.O(\mathbf{problems}'))((\lambda f.\mathbf{if}'(D(f(\mathbf{three}'(\mathbf{workers}')))(\lambda y.\exists g[CH(g) \wedge g(\mathbf{baby}')(\lambda x.\mathbf{have}'(x)(y))])))(h)))_{c\to t}}\text{RCOND}^1 \text{ (i)}}{(\mathbf{E}(CH)(\lambda h.(\lambda O.O(\mathbf{problems}'))((\lambda f.\mathbf{if}'(D(f(\mathbf{three}'(\mathbf{workers}')))(\lambda y.\exists g[CH(g) \wedge g(\mathbf{baby}')(\lambda x.\mathbf{have}'(x)(y))])))(h))))_t}\text{SAT (ii)}$$

simplification and renaming:

$$= \exists f[CH(f) \wedge \mathbf{if}'(D(f(\mathbf{three}'(\mathbf{workers}')))(\lambda y.\exists g[CH(g) \wedge g(\mathbf{baby}')(\lambda x.\mathbf{have}'(x)(y))]))(\mathbf{problems}')]$$

By (i)+(ii) this proposition is in $clos_1(TR_B(\mathrm{S})) \subseteq TR(\mathrm{S})$.
Thus, it is also in the complete translation set of the tree.

References

Abusch, D. (1994), "The scope of indefinites", *Natural Language Semantics* 3: 88-135.

Barwise, J., and R. Cooper (1981), "Generalized quantifiers and natural language", *Linguistics and Philosophy* 4: 159–219.

Beghelli, F., "A minimalist approach to quantifier scope", in *Proceedings of NELS23*.

Ben-Shalom, D., "Object wide scope and semantic trees", in *Proceedings of SALT3*.

Bennett, M.R. (1974), *Some Extensions of a Montague Fragment of English*, Ph.D. dissertation, UCLA.

van Benthem, J. (1986), *Essays in Logical Semantics*, Reidel, Dordrecht.

van Benthem, J. (1991), *Language in Action*, North-Holland.

Carlson, G. (1977), *Reference to Kinds in English*, Ph.D. dissertation, UMASS.

Carpenter, B. (1994a), "Distribution, collection and quantification: a type-logical account of plurality", to appear in B. Carpenter, *Lectures on Type Logical Semantics*, MIT Press.

Carpenter, B. (1994b), "Quantification and scoping: a deductive account", to appear in B. Carpenter, *Lectures on Type Logical Semantics*, MIT Press.

Chierchia, G. (1993), "Questions with quantifiers", *Natural Language Semantics* 1:181-234.

Chomsky, N. (1986), *Barriers*, MIT Press, Cambridge, Mass.

Cooper, R. (1975), *Montague's Semantic Theory and Transformational Syntax*, Ph.D. dissertation, UMASS.

van der Does, J. (1992), *Applied Quantifier Logics*, Ph.D. dissertation, University of Amsterdam.

van der Does, J. (1993), "Sums and quantifiers", *Linguistics and Philosophy* 16: 509-550.

Dowty, D. (1986), "A note on collective predicates, distributive predicates and *all*", in Marshall, Miller and Zhang(eds.), *Proceedings of the Third Eastern States Conference on Linguistics*, Ohio State University, Columbus.

Egli, U., and K. von Heusinger (1995), "The epsilon operator and e-type pronouns", in U. Egli, P.E. Pause, C. Schwarze, A. von Stechow and G. Wienold(eds), *Lexical Knowledge in the Organization of Language*, John Benjamins, Amsterdam-Philadelphia.

Farkas, D.F. (1981), “Quantifier scope and syntactic islands”, in *Proceedings of CLS 17*.
Farkas, D.F. (1995), “Evaluation indices and scope”, to appear in A. Szabolcsi(ed.), *Ways of Scope Taking*, Kluwer.
Fine, K. (1985), *Reasoning with Arbitrary Objects*, Blackwell, Oxford.
Fodor, J.D., and I. Sag (1982), “Referential and quantificational indefinites”, *Linguistics and Philosophy* 5:355-398.
Fox, D. (1995), “Economy and scope”, *Natural Language Semantics* 3.
Heim, I., H. Lasnik and R. May (1991), “Reciprocity and plurality”, *Linguistic Inquiry* 22: 63-101.
Hendriks, H. (1993), *Studied Flexibility*, Ph.D. dissertation, University of Amsterdam.
Hepple, M. (1991), *The Grammar and Processing of Order and Dependency: A Categorial Approach*, Ph.D. Dissertation, University of Edinburgh.
Higginbotham, J. (1987), “Indefiniteness and predication”, in E.J. Reuland and A.G.B. ter Meulen(eds.), *The Representation of (In)definiteness*, MIT Press.
Hilbert, D., and P. Bernays (1939), *Die Grundlagen der Mathematik II*, second edition, reprint 1970, Springer, Berlin.
Hoeksema, J. (1983), “Plurality and conjunction”, in A. ter Meulen(ed.), *Studies in Modeltheoretic Semantics*, Foris, Dordrecht.
Jacobson, P. (1992), “Bach-Peters Sentences in a Variable-Free Semantics”, in P. Dekker and M. Stokhof(eds.), *Proceedings of the Eighth Amsterdam Colloquium*.
Kamp, H., and U. Reyle (1993), *From Discourse to Logic*, Kluwer.
Keenan, E. (1987), “A semantic definition of ‘indefinite NP”’, in E.J. Reuland and A.G.B. ter Meulen(eds.), *The Representation of (In)definiteness*, MIT Press.
Keenan, E., and L. Faltz (1985), *Boolean Semantics for Natural Language*, Reidel, Dordrecht.
Kratzer, A. (1995), “Scope or pseudoscope? Are there wide scope indefinites”, an unpublished ms., UMASS. To appear in F. Hamm and A. von Stechow(eds.), proceedings of *Recent Developments in the Theory of Natural Language*, Universität Tübingen.
Lakoff, G. (1970), “Repartee”, *Foundations of Language* 6:389-422.
Landman, F. (1989), “Groups I&II”, *Linguistics and Philosophy* 12: 559-605,723-744.
Link, G. (1983), “The logical analysis of plurals and mass terms: a lattice theoretical approach”, in R. Bauerle et al.(eds.), *Meaning, Use and Interpretation of Language*, De Gruyter, Berlin.
Link, G. (1987), “Generalized quantifiers and plurals”, in P. Gärdenfors(ed.), *Generalized Quantifiers*, D. Reidel.
Ludlow, P., and S. Neale (1991), “Indefinite descriptions: in defense of Russell”, *Linguistics and Philosophy* 14:171-202.
May, R. (1977), *The Grammar of Quantification*, Ph.D. dissertation, MIT.
Meyer Viol, W.P.M. (1995), *Instantial Logic*, Ph.D. dissertation, Utrecht University, ILLC dissertation series, Amsterdam.
Moortgat, M. (1988), *Categorial Investigations, Logical and Linguistic Aspects of the Lambek Calculus*, Foris, Dordrecht.
Nam, S. (1991), “N-ary quantifiers and the expressive power of NP compositions”, in J. van der Does and J. van Eijck(eds.), *Generalized Quantifiers Theory and Application*, Dutch network for language, logic and information.
Partee, B. (1987), “Noun phrase interpretation and type shifting principles”, in J. Groenendijk, D. de Jongh and M. Stokhof(eds.), *Studies in Discourse Representation Theories and the Theory of Generalized Quantifiers*, Foris, Dordrecht.
Partee, B., and M. Rooth (1983), “Generalized conjunction and type ambiguity”, in R. Bauerle et al.(eds.), *Meaning, Use and Interpretation of Language*, De Gruyter, Berlin.

Quine, W.V., “Variables explained away”, in *Selected Logic Papers*, Random House, New York.

Reinhart, T. (1992), “Wh-in-situ: an apparent paradox”, in P. Dekker and M. Stokhof(eds.), *Proceedings of the Eighth Amsterdam Colloquium*.

Reinhart, T. (1996), “Quantifier scope”, to appear in *Linguistics and Philosophy*. Prepublished in T. Reinhart, *Interface Strategies*, OTS Working Papers 95-002, Utrecht University.

Rodman, R. (1976), “Scope phenomena, ‘movement transformations’ and relative clauses”, in B.H. Partee(ed.), *Montague Grammar*, Academic Press, New York.

Rooth, M., and B. Partee (1982), “Conjunction, type ambiguity and wide scope *or*”, in Flickinger et al.(eds.), *Proceedings of the First West Coast Conference on Formal Linguistics*, Stanford.

Ross, J.R. (1967), *Constraints on Variables in Syntax*, Ph.D. dissertation, MIT.

Ruys, E.G. (1992), *The Scope of Indefinites*, Ph.D. dissertation, Utrecht University.

Ruys, E.G. (1995), “Weak Crossover as a scope phenomenon”, unpublished ms., Utrecht University.

Scha, R. (1981), “Distributive, collective and cumulative quantification”, in J. Groenendijk, M. Stokhof and T.M.V. Janssen(eds.), *Formal Methods in the Study of Language*, Amsterdam.

Schein, B. (1993), *Plurals and Events*, MIT Press.

Szabolcsi, A. (1987), “Bound variables in syntax: are there any?”, in J. Groenendijk et al.(eds.), *Proceedings of the Sixth Amsterdam Colloquium*.

Szabolcsi, A. (1996), “Strategies for scope taking”, to appear in A. Szabolcsi(ed.), *Ways of Scope Taking*, Kluwer.

Verkuyl, H.J. (1993), *A Theory of Aspectuality*, Cambridge University Press.

Winter, Y. (1995a), “Syncategorematic conjunction and structured meanings”, in *Proceedings of SALT5*.

Winter, Y. (1995b), “On the formalization of choice functions as representing the scope of indefinites”, in G.V. Morrill and R.T. Oehrle(eds.), *Formal Grammar*, proceedings of the conference of the European summer school in Logic, Language and Information, Barcelona.

Winter, Y. (1996a), “A unified semantic treatment of singular NP coordination”, *Linguistics and Philosophy* 19:337-391.

Winter, Y. (1996b), “The Square of Individuals”, in P. Dekker and J. Groenendijk(eds.), *Proceedings of the Tenth Amsterdam Colloquium*.

Zimmermann, T.E. (1986), “Transparent adverbs and scopeless quantifiers”, in J. Groenendijk, D. de Jongh and M. Stokhof(eds.), *Foundations of Pragmatics and Lexical Semantics*, Foris.

Zimmermann, T.E. (1991), “Scopeless quantifiers and operators”, in J. van der Does and J. van Eijck(eds.), *Generalized Quantifiers Theory and Application*, Dutch network for language, logic and information.

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