Monotonicity and Collective Quantification

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Abstract

This article studies the monotonicity behavior of plural determiners that quantify over collections. Following previous work, we describe the collective interpretation of determiners such as *all*, *some* and *most* using generalized quantifiers of a higher type that are obtained systematically by applying a type shifting operator to the standard meanings of determiners in Generalized Quantifier Theory. Two processes of *counting* and *existential quantification* that appear with plural quantifiers are unified into a single *determiner fitting* operator, which, unlike previous proposals, both captures existential quantification with plural determiners and respects their monotonicity properties. However, some previously unnoticed facts indicate that monotonicity of plural determiners is not always preserved when they apply to collective predicates. We show that the proposed operator describes this behavior correctly, and characterize the monotonicity of the collective determiners it derives. It is proved that determiner fitting always preserves monotonicity properties of determiners in their second argument, but monotonicity in the first argument of a determiner is preserved if and only if it is monotonic in the same direction in the second argument. We argue that this asymmetry follows from the *conservativity* of generalized quantifiers in natural language.

Keywords: collectivity, determiner, generalized quantifier, monotonicity, plural, type shifting.

1 Introduction

Traditional logical studies of quantification in natural language concentrated on the interactions between quantifiers and *distributive predicates* – predicates that describe properties of atomic entities. *Generalized Quantifier Theory* (GQT), as was applied to natural language semantics in the influential works of Barwise and Cooper (1981), Van Benthem (1984) and Keenan and Stavi (1986), followed this tradition and concentrated on 'atomic' quantification. The framework that emerges from these works provides a general treatment of sentences such as the following.

(1) *All the students are happy. Some girls arrived. No pilot is hungry. Most teachers are Republican. Exactly five boys smiled. Not all the children sneezed.*

In these sentences, the denotations of both the nominal (e.g. *students, girls*, etc.) and the verb phrase (e.g. *be happy, arrived*, etc.) are traditionally treated as distributive predicates, which correspond to subsets of a domain of (arbitrary) atomic entities. Standard GQT assigns determiners such as *all, some* and *most* denotations that are relations between such sets of atomic entities.

While this general treatment is well-motivated, it does not account for the interactions between quantifiers and *collective* predicates. Consider for instance the following sentences.

(2) *All the colleagues cooperated. Some girls sang together. No pilots dispersed. Most of the sisters saw each other. Exactly five friends met at the restaurant. Not all the children gathered.*

According to most theories of plurals, nominals such as *colleagues, sisters* and *friends* and verb phrases such as *cooperated, gathered* and *saw each other* do not denote sets of atomic entities, but rather sets of *collections* of such entities. There are various theories about the algebraic structure of such collections, but for our purposes in this article it is sufficient to assume that collections are *sets* of atomic entities. Thus, we assume that collective predicates denote *sets of sets* of atomic entities. Consequently, the standard denotation of determiners in GQT as relations between sets of atoms is not directly applicable to sentences with collective predicates.

Early contributions to the study of collective quantification in natural language, most notably Scha (1981), propose that meanings of 'collective statements' as in (2) are derived using 'collective' denotations of determiners.[1] More recent works, including among others Van der Does (1992,1993), Dalrymple et al. (1998) and Winter (1998,2001), , propose to derive such collective meanings of determiners from their standard distributive denotations in GQT using general mappings that apply to these distributive meanings. In the works of Van der Does and Winter, *type shifting operators* apply to a standard determiner denotation D, which ranges over atomic entities, and derives a determiner of a higher type $O(D)$, which ranges over sets of atomic entities.[2] The study of collective quantification as in (2) is reduced in these theories to the study of the available O mapping(s) from standard determiners to determiners over collections. We follow Winter (2001) and adopt one general type shifting principle for collective quantification that unifies Scha's and Van der Does' 'neutral' and 'existential' liftings of determiners into one operator. This operator is referred to as *determiner fitting* (*dfit*).

[1]For earlier works on plural quantification within Montague Grammar see Bennett (1974) and Hauser (1974).

[2]As we shall see below, the *bounded composition* operator that Dalrymple et al. propose can also be cast to a type shifting operator on determiners.

The type shifting approach establishes a connection between standard GQT and linguistic theories of plurality. A natural question that arises in this context is: what are the relations between semantic properties of standard quantifiers in GQT and properties of their 'collectivized' version? In this article we concentrate on the *monotonicity* properties of determiners, which, as far as standard distributive quantification is concerned, are one of the best studied aspects of quantification in natural language.[3] Consider for instance the simple valid entailments (denoted by '$\Rightarrow$') in (3), with the determiner *all*.

(3) a. All the students are happy $\Rightarrow$ All the rich students are happy.

b. All the students are very happy $\Rightarrow$ All the students are happy.

Intuitively, the entailments in (3a-b) show that, in simple sentences, the determiner *all* licenses a replacement of its first argument (*students*) by any *subset* of this argument (e.g. *rich students*), and licenses a replacement of its second argument (*very happy*) by any *superset* of this argument (e.g. *happy*). Thus, the determiner *all* is classified as *downward monotone* in its first argument but *upward monotone* in its second argument.

The starting point for the investigations in this work is the observation that such monotonicity entailments are not always preserved when the determiner quantifies over collections. Consider for instance the contrast between the sound entailment in (3a) and the invalid entailment in (4) below.

(4) All the students drank a whole glass of beer together $\not\Rightarrow$ All the rich students drank a whole glass of beer together.

In a situation where the students are s_1, s_2 and s_3 and the rich students are s_1 and s_2, assume that the group $\{s_1, s_2, s_3\}$ drank a whole glass of beer together, but no other group did. In this situation, the antecedent in (4) is obviously true, but the consequent is false. However, as we shall see, many other plural determiners do not lose their monotonicity properties when they apply to collective predicates. This variation calls for a systematic account of the monotonicity properties of determiners in their collective usage, in relation to their monotonicity properties in standard GQT.

The aim of this work is to study these relations in detail. We will prove that, under the adopted determiner fitting operator, 'monotonicity loss' with *all* is strongly

[3] See Ladusaw (1979), Fauconnier (1978) and much recent work on the linguistic centrality of monotonicity for describing the distribution of negative polarity items like *any* or *ever*. For instance, in correlation with the monotonicity properties of *all* as reflected in (3), the negative polarity item *ever* can appear in the nominal argument of *all* (e.g. in (i) below), in which it is downward monotone, but not in the verb phrase argument (e.g. in (ii)), where *all* is upward monotone:

(i) All the [students who have ever visited Haifa][came to the meeting].

(ii) $*$All the [students who came to the meeting][have ever visited Haifa].

connected to the fact that the monotonicity properties of this determiner are different in its two arguments. We show that determiner fitting preserves the monotonicity properties of determiners in their second argument, and it further preserves monotonicity properties of determiners which have the same monotonicity properties in both arguments. However, with determiners such as *all*, *not all*, *some but not all*, and *either all or none (of the)*, which are monotone in their first argument, but have a different monotonicity property in the second argument, monotonicity in the first argument is not preserved under determiner fitting. We claim that the origin of these (empirically welcome) results is in the 'neutral' process that Scha proposed for collective quantification, and that the combination of this treatment with an 'existential' lifting, which is empirically well-motivated, has no effects whatsoever on the (non-)preservation of (non-)monotonicity with collective quantifiers.

The structure of the rest of this article is as follows. Section 2 reviews some familiar notions from GQT that are used in subsequent sections. Section 3 describes previous treatments of collective quantification and the uniform type shifting strategy that is adopted in this paper. Section 4 establishes the facts pertaining to (non-)preservation of (non-)monotonicity under type shifting with all possible monotonicity properties of determiners in GQT.

2 Notions from generalized quantifier theory

This section reviews some familiar notions from standard GQT that are important for the developments in subsequent sections. For an exhaustive survey of standard GQT see Keenan and Westerståhl (1996).

The main property of quantifiers that is studied in this article is *monotonicity*, which is a general concept that describes 'order preserving' properties of functions over partially ordered domains.

Definition 1 (monotonicity) *Let $\langle A_1, \leq_1\rangle, \ldots, \langle A_n, \leq_n\rangle$ and $\langle B, \leq_B\rangle$ be partially ordered sets, and let f be a function from $A_1 \times ... \times A_n$ to B. The function f is called* upward (downward) monotone *in its i-th argument iff for all $a_1 \in A_1, \ldots, a_n \in A_n$: if $a'_i \in A_i$ satisfies $a_i \leq_i a'_i$ ($a'_i \leq_i a_i$) then $f(a_1, \ldots, a_i, \ldots, a_n) \leq_B f(a_1, \ldots, a'_i, \ldots, a_n)$.*
We say that f is monotone *in its i-th argument iff f is either upward or downward monotone in its i-th argument.*

Extensional denotations are given relative to an arbitrary non-empty finite set E, to which we refer as the domain of *atomic entities*, or simply *atoms*. Given a non-empty domain E, a *determiner* over E is a function from $\wp(E) \times \wp(E)$ to $\{0, 1\}$.[4] Hence, a determiner is a relation between subsets of E. The set $\wp(E)$, the power set of E, is ordered by set inclusion. The set $\{0, 1\}$, the domain of *truth values*, is ordered by implication, which is simply the numerical '$\leq$' order on

[4]Later in the paper, we refer to such determiners as *Atom-Atom determiners*, since both their arguments are sets of atomic entities.

Determiner	Denotation: for all $A, B \subseteq E$:	Monotonicity
all	$\mathbf{all}'(A)(B) = 1 \Leftrightarrow A \subseteq B$	$\downarrow$MON$\uparrow$
not all	$(\neg\mathbf{all}')(A)(B) = 1 \Leftrightarrow A \setminus B \neq \emptyset$	$\uparrow$MON$\downarrow$
some	$\mathbf{some}'(A)(B) = 1 \Leftrightarrow A \cap B \neq \emptyset$	$\uparrow$MON$\uparrow$
no	$\mathbf{no}'(A)(B) = 1 \Leftrightarrow A \cap B = \emptyset$	$\downarrow$MON$\downarrow$
most	$\mathbf{most}'(A)(B) = 1 \Leftrightarrow \lvert A \cap B\rvert > \lvert A \setminus B\rvert$	$\sim$MON$\uparrow$
exactly five	$\mathbf{exactly\_5}'(A)(B) = 1 \Leftrightarrow \lvert A \cap B\rvert = 5$	$\sim$MON$\sim$

Table 1: standard denotations of some determiners

$\{0, 1\}$. Since a determiner is a two-place function, we use the terms *left monotonicity* and *right monotonicity* for referring to its monotonicity in the first and second arguments respectively. We use the following notation:

- $\uparrow$MON, $\downarrow$MON and $\sim$MON for determiners that are upward left-monotone, downward left-monotone and not left-monotone, respectively.
- MON$\uparrow$, MON$\downarrow$ and MON$\sim$ for determiners that are upward right-monotone, downward right-monotone and not right-monotone, respectively.

We combine these two notations, and say for instance that the determiner *all* is $\downarrow$MON$\uparrow$ according to its definition as the subset relation. The standard denotations that are assumed for some determiners are given in table 1, together with their monotonicity properties.

The well-known *conservativity* property of determiners reflects the observation that the truth value that determiners in natural language assign to any pair of sets A and B is identical to the truth value they assign to A and $A \cap B$. This is observed in (seemingly obvious) equivalences such as the following.

(5) All the/some/no/most of the/exactly five cars are blue $\Leftrightarrow$
All the/some/no/most of the/exactly five cars are blue cars.

Formally, conservativity of determiner functions is defined as follows.

Definition 2 (conservativity) *A determiner D over E is* conservative *(CONS) iff for all $A, B \subseteq E$: $D(A)(B) = D(A)(A \cap B)$.*

Thus, in order to evaluate the truth value of a sentence *D students are hungry*, with a conservative determiner D, we do not have to know the set of all hungry entities, but only the set of hungry *students*.

Let us mention a useful constancy aspect of the meaning of most natural language determiners.[5] Van Benthem (1984) defines the *permutation invariant* (PI) determiners as follows.

[5]The exceptions to PI are determiners such as *my*, *her* or *this*, which we henceforth ignore.

Definition 3 (permutation invariance) *A determiner D over E is* permutation invariant *(PI) iff for all permutations π of E and for all $A, B \subseteq E$: $D(\{\pi(x) : x \in A\})(\{\pi(y) : y \in B\}) = D(A)(B)$.*

Roughly speaking, when a determiner D is PI, this means that it is not sensitive to the identity of the members in its arguments, but only to set-theoretical relations between its arguments. It is easy to verify that the determiners in table 1 are all PI.

The following two definitions characterize two trivial classes of determiners.

Definition 4 (left/right triviality) *A determiner D over E is called* left (right) trivial *iff for all $A, B, C \subseteq E$: $D(A)(B) = D(C)(B)$ ($D(A)(B) = D(A)(C)$ respectively).*

Intuitively, an LTRIV (RTRIV) determiner is insensitive to the identity of its left (right) argument. For instance, the determiners *less than zero* and *at least zero*, which are both LTRIV and RTRIV, assign the same truth value (0 and 1 respectively) to all possible arguments. We occasionally restrict our attention to determiners that are not right trivial. This is because non-right-triviality is a stronger restriction on conservative determiners than non-left-triviality — provably, all conservative LTRIV determiners are RTRIV. However, a conservative RTRIV determiner is not necessarily LTRIV. For instance, the determiner D s.t. $D(A)(B) = 1$ iff $A \neq \emptyset$ is conservative and RTRIV but is not LTRIV.

In this article we study the monotonicity properties of non-right-trivial conservative determiners that satisfy permutation invariance, which are also in the main focus of the general theory of quantification in natural language.

3 Collective determiners and type shifting principles

The type shifting account of plural determiners that was initiated by Scha (1981) is motivated by sentences such as those in (2), which involve collective predicates. In this section we review previous treatments of collective quantification, and concentrate on the proposal of Winter (1998,2001) that aims to solve some of the empirical problems for Scha's and Van der Does' proposals. As an example that illustrates many aspects of the interpretation of collective determiners, consider the following sentence.

(6) Exactly five students drank a whole glass of beer together.

The denotation of the collective predicate *drank a whole glass of beer together* is assumed to be an element of $\wp(\wp(E))$ – a set of sets of atomic elements. To interpret sentence (6), the meaning of the determiner *exactly five* from table 1 is shifted so that it can combine with this collective predicate. This section deals with the proper way(s) to define such a shifting operator.

3.1 The type shifting operators of Scha and Van der Does

Scha (1981) proposes an extension of standard GQT to the treatment of collectivity phenomena as in sentence (6). The work in Van der Does (1992,1993) contains a systematic reformulation of Scha's approach using type shifting operators within contemporary GQT. In the systems of Scha and Van der Does (henceforth S&D), both distributive and collective verbal predicates (e.g. *smile, meet*) denote elements of $\wp(\wp(E))$. However, nominal predicates such as *students* standardly denote subsets of E. Accordingly, in S&D's proposal, collective determiners are functions from $\wp(E) \times \wp(\wp(E))$ to $\{0, 1\}$. We distinguish such *Atom-Set* determiners, where the first argument is a set of atoms and the second argument is a set of sets of atoms, from the standard *Atom-Atom* determiners of GQT as in table 1, where both arguments are sets of atoms. Van der Does follows Van Benthem (1991:68) and proposes that the Atom-Set determiners that are necessary for the interpretation of plural sentences can be obtained systematically from Atom-Atom determiners. There are two collective shifts that are proposed for sentences like (6) in S&D's works.[6] One collective operator is called **E** (for *Existential operator*). For sentence (6) this operator generates a statement that claims that there is a set of exactly five students and that this set drank a whole glass of beer together. In general, for any Atom-Atom determiner D over a domain E, applying the **E** operator leads to the Atom-Set determiner $\mathbf{E}(D)$ which is defined for all $A \subseteq E$ and $\mathcal{B} \subseteq \wp(E)$ by:

$$(\mathbf{E}(D))(A)(\mathcal{B}) = 1 \;\Leftrightarrow\; \exists X \in \mathcal{B} \cap \wp(A)[D(A)(X) = 1].$$

Scha proposes another collective analysis of plural determiners, which Van der Does refers to as *neutral*. In sentence (6), for instance, this neutral analysis counts all the individual students who participated in sets of students that drank a whole glass of beer together, and requires that the total number of these students is exactly five. For any Atom-Atom determiner D over E, the corresponding Atom-Set determiner $\mathbf{N}(D)$ (Van der Does' $\mathbf{N}_2$) is defined for all $A \subseteq E$ and $\mathcal{B} \subseteq \wp(E)$ by:

$$(\mathbf{N}(D))(A)(\mathcal{B}) \overset{def}{=} D(A)(\cup(\mathcal{B} \cap \wp(A)).$$

Note that the set $\cup(\mathcal{B} \cap \wp(A))$ contains x if and only if x is an element of a subset of A that belongs to $\mathcal{B}$.

The **N** operator involves a mapping of the left argument A of the determiner, which is a set of atoms, into the *power set* of A, which is a set of sets of atoms. Such a mapping from sets to sets of sets is useful in S&D's strategy, as in most other theories of plurals, since it makes a connection between distributive predicates and collective predicates. This is required whenever a predicate over atoms semantically interacts with other elements that range over collections (e.g. sets). In the theory of plurals, such a mapping is often referred to as a *distributivity operator*. The power set operator $\wp$ is sufficient as a distributivity operator for our

[6] S&D also assume a distributive shift, which is irrelevant for our purposes here. In addition, Van der Does (1992) considers a third collective shift but (inconclusively) dismisses it in his 1993 article.

purposes in this paper.[7] We say that a set $\mathcal{A} \subseteq \wp(E)$ is a *distributed set of atoms* (DSA) if $\mathcal{A} = \wp(A)$ for some set of atoms $A \subseteq E$.

In sentences where the second argument of the determiner is an ordinary distributive predicate, its meaning under S&D's treatment can be defined as a distributed set of atoms rather than a set of atoms. This is needed in order to match the type of the second argument of the lifted Atom-Set determiner. For instance, the standard meaning of sentence (7) below is captured using the **N** operator as in (8), and not simply by directly applying the Atom-Atom denotation of the determiner *exactly five* to two sets of atoms, as in standard GQT.

(7) Exactly five students sang.

(8) $(\mathbf{N}(\mathbf{exactly\_5'}))(\mathbf{student'})(\wp(\mathbf{sing'}))$,
where $\mathbf{student'}, \mathbf{sing'} \subseteq E$.

It is easy to verify that this analysis is equivalent to the standard analysis of (7). More generally, observe the following fact.

Fact 1 *For every conservative Atom-Atom determiner D over E, for all $A, B \subseteq E$: $(\mathbf{N}(D))(A)(\wp(B)) = D(A)(B)$.*

3.2 Problems for S&D's strategies

One empirical problem for S&D's type shifting analysis follows from a warning in Van Benthem (1986:52-53), and is accordingly referred to as the *Van Benthem problem* for plural quantification. Van Benthem mentions that any general existential lifting such as the **E** operator is problematic, because it turns any Atom-Atom determiner into an Atom-Set determiner that is upward right-monotone.

Quite expectedly, this property of the **E** operator is empirically problematic with many Atom-Atom determiners that are not upward right monotone. For instance, using the **E** operator, sentence (9) below, with the $\mathrm{MON}\!\downarrow$ determiner *no*, gets the interpretation in (10).

(9) No students met yesterday at the coffee shop.

(10) $(\mathbf{E}(\mathbf{no'}))(\mathbf{student'})(\mathbf{meet\_yesterday'}) = 1$
$\Leftrightarrow\ \emptyset \in \mathbf{meet\_yesterday'}$

This analysis of sentence (9) makes the strange claim that an empty set met yesterday at the coffee shop, which is clearly not what sentence (9) claims.[8] The

[7] A more common version of a distributivity operator is the $\wp^+$ operator, which maps each set to its power set minus the empty set: $\wp^+(A) = \wp(A) \setminus \{\emptyset\}$. Using the $\wp^+$ operator would not change the results in this paper, and therefore we use the simpler power set operator. For arguments in favor of a distributivity operator that is a more sophisticated than $\wp^+$ see Schwarzschild (1996). For counterarguments see Winter (2000).

[8] Using $\wp^+$ instead of $\wp$ in the definition of the **E** operator (which is what S&D do), sentence (9) is analyzed as a contradiction. Obviously, this is not the correct analysis of the sentence either.

existential analysis reverses the monotonicity properties of the determiner *no*, so that $\mathbf{E}(\mathbf{no}')$ is ↑MON↑. However, the determiner *no* remains ↓MON↓ in this case even though it is used for quantification over collections. For instance, sentence (9) entails sentence (11a) below and does not entail sentence (11b).

(11) a. No tall students met yesterday evening at the coffee shop.

b. No people (ever) met at the coffee shop.

The problem is manifested even more dramatically when the second argument of the determiner is a distributive predicate (distributed by $\wp$). For instance, sentence (12) below is analyzed as in (13), which is a tautology (for choose $X = \emptyset$).[9]

(12) Less than five students smiled.

(13) $(\mathbf{E}(\mathbf{less\_than\_5}'))(\mathbf{student}')(\wp(\mathbf{smile}')) = 1$
$\Leftrightarrow\ \exists X[X \subseteq \mathbf{smile}' \wedge X \subseteq \mathbf{student}' \wedge |X| < 5]$

Although the Van Benthem problem indicates that the existential operator is inadequate, this operator still captures one effect that the **N** operator by itself does not handle. To see that, reconsider sentence (6), restated in (14) below, and its analyses using the **N** and **E** operators.

(14) Exactly five students drank a whole glass of beer together.

a. $(\mathbf{N}(\mathbf{exactly\_5}'))(\mathbf{student}')(\mathbf{drink\_beer}') = 1$
$\Leftrightarrow\ |\{x \in E : \exists A \subseteq \mathbf{student}'[x \in A \wedge \mathbf{drink\_beer}'(A)]\}| = 5$

b. $(\mathbf{E}(\mathbf{exactly\_5}'))(\mathbf{student}')(\mathbf{drink\_beer}') = 1$
$\Leftrightarrow\ \exists A \subseteq \mathbf{student}'[|A| = 5 \wedge \mathbf{drink\_beer}'(A)]$

The analysis in (14a) requires that the total number of students in sets of students that drank a glass of beer together is five. However, in addition, sentence (14) also requires that there was a set of five students who drank a whole glass of beer together. The **E** operator in the analysis in (14b) imposes this requirement, but fails to take into account the total number of students who drank a glass of beer, and therefore leads to the Van Benthem problem. A similar dilemma arises with upward monotone determiners, as in the following example.

(15) More than five students drank a whole glass of beer together.

In this case too, the **N** operator imposes a requirement only on the total number of students involved in beer drinking events, whereas what we need in this case is an existential reading, requiring that there actually was a set with more than five students who drank a whole glass of beer together. In S&D's systems there

[9] If we replace $\wp$ by $\wp^+$, then sentence (12) is analyzed as being equivalent to *at least one student smiled*, which is bad enough.

is no clear specification of how to capture both aspects of collective quantification without generating undesired truth conditions.[10]

Another problem for S&D's strategy is in the type of collective determiners it assumes. In S&D's proposal, any collective determiner is an Atom-Set determiner. However, in many cases, a collective predicate may also appear in the left argument of a determiner. For instance, reconsider the following example from (2).

(16) All the colleagues cooperated.

In this case, the plural noun *colleagues* is collective: to say that a and b are colleagues is not the same as saying that a is a colleague and b is a colleague. Other collective nouns like *brothers, sisters, friends* etc. lead to similar problems. Other cases where the first argument of a determiner is collective appear when a distributive noun is modified by a collective predicate. For instance, consider the following examples.

(17) Exactly four similar students smiled.

(18) Most of the students who saw each other played chess.

In these cases, the interpretation of the first argument involves intersection of a distributive predicate (distributed by $\wp$) with a collective predicate. For instance, the denotation of the nominal *similar students* in (17) is obtained by intersecting the set of sets of students with the set of sets of similar entities. These examples indicate that collective determiners should allow collective predicates in both arguments, and not only in the right argument as in S&D's lifting strategies.

3.3 Dalrymple et al.'s bounded composition operator

Dalrymple et al. (1998) concentrate on the semantics of reciprocal expressions (*each other*, *one another*) in sentences with simple plural NPs such as *the children* and *Mary and John*. However, they also address the problem of interpreting reciprocal expressions in the following sentences, where a collective reciprocal predicate combines with a quantifier of more complex monotonicity properties.

(19) a. Many people are familiar to one another.
     b. Most couples in the apartment complex babysit for each other.
     c. At most five men hit each other.

Dalrymple et al. observe the existential requirement in sentences (19a-b), and their treatment of such sentences, with $\mathrm{MON}\uparrow$ determiners, is accordingly a reformulation of the **E** operator of Scha and Van der Does. However, to overcome the problems that the existential requirement creates in sentences such as (19c), with

[10] Van der Does (1992) tries to overcome this problem by proposing a syntactic mechanism of feature propagation that is designed to rule out some of the undesired effects of his semantic system.

non-$\mathrm{MON}\uparrow$ determiners, Dalrymple et al. use a different analysis for such sentences. In example (19c), their analysis requires that each set of men who hit each other contains at most five men. It seems quite likely that for the case of sentence (19c), this analysis reflects a possible reading.[11]

Dalrymple et al. combine the two processes they assume for $\mathrm{MON}\uparrow$ and non-$\mathrm{MON}\uparrow$ determiners into one general operator that they call *Bounded Composition*. This operator can be cast as a lifting operator of determiners, so that for any Atom-Atom determiner D over E, the corresponding Atom-Set determiner $\mathbf{BC}(D)$ is defined for all $A \subseteq E$ and $\mathcal{B} \subseteq \wp(E)$ by:[12]

$$
\begin{array}{l}
(\mathbf{BC}(D))(A)(\mathcal{B}) = 1 \textit{ iff} \\
\quad \forall Y \in \wp(A) \cap \mathcal{B}\ \exists X \in \wp(A) \cap \mathcal{B} \\
\qquad\qquad [|X| \geq |Y| \wedge |A \setminus X| \leq |A \setminus Y| \wedge D(A)(X) = 1] \\
\quad \wedge(\exists X \in \wp(A) \cap \mathcal{B}\ [D(A)(X) = 1] \vee D(A)(\emptyset) = 1).
\end{array}
$$

The first conjunct in this definition reflects a counting process, parallel to S&D's **N** operator.[13] The second conjunct adds to this process an existential requirement, similar to S&D's **E** operator. However, there are two modifications in the usage of these two processes, compared to S&D's strategies:

1. Unlike the **N** operator, the counting process within **BC** does not require the total union of $\wp(A) \cap \mathcal{B}$ to be in the generalized quantifier $D(A)$, but only requires that each set of maximal cardinality within $\wp(A) \cap \mathcal{B}$ is in $D(A)$.

2. The existential requirement overcomes Van Benthem's problem, due to the disjunct $D(A)(\emptyset) = 1$ within it, which properly weakens the **E** operator with determiners that satisfy $D(A)(\emptyset) = 1$.

The motivation for the introduction of the bounded composition operator is to treat collective readings of GQs with reciprocal predicates. However, we believe that the combination of counting and existential processes is a promising aspect of Dalrymple et al.'s proposal also for other cases of collectivity. On the other hand, the empirical adequacy of the counting process as implemented within the **BC** operator is not completely clear to us. Some speakers we consulted accept Dalrymple et al.'s assumption that sentences such as (19c), with a downward monotone quantifier, can be true even though the 'total' set of people who participated in sets of

[11] Dalrymple et al.'s intuitions about the meaning of (19c) seem to be similar to those of Schein (1993), who proposes an event semantics of plurals. For some remarks on the empirical question concerning the generality of this analysis see our discussion below.

[12] The operator that Dalrymple et al. propose is defined as a 4-ary relation between a determiner, a set of atoms, a binary relation and the meaning of the reciprocal expression. For instance, in sentence (19a), these are (respectively) the meanings of the expressions *many*, *people*, *familiar to* and *one another*. For our purposes it is sufficient to consider the determiner alone, because the compositional interpretation of reciprocal predicates such as *familiar to one another* is not in the focus of this article.

[13] The requirement $|A \setminus X| \leq |A \setminus Y|$ in this conjunct is needed only when we assume infinite domains. Over finite domains it follows from the requirement $|X| \geq |Y|$.

people who hit each other is not in the quantifier (i.e. in this case – includes more than five members). However, these judgments did not seem to be highly robust and they vary considerably when the determiner *at most five* is replaced by other non-$\mathrm{MON}\uparrow$ determiners such as *less than five, exactly five* or *between five and ten*. For example, consider the following sentence.

(20) Exactly five students hit each other.

Assume that there was a set of exactly five students who hit each other, and that in addition there was only one other set of students A who hit each other. In case there are four students in A, then the **BC** operator renders sentence (20) true. However, if A contains six students then the **BC** operator takes sentence (20) to be false. We did not trace such a difference in our informants' intuitions about the sentence.

As a general operation for deriving collective readings of GQs, the **BC** operator shows some undesired effects when the quantifier it derives interacts with a so-called 'mixed' predicate. These predicates (unlike predicates formed with reciprocals) can also be true of singleton sets, in addition to sets with two or more elements. Consider for example the following sentence.

(21) At most five students drank a whole glass of beer (together or separately).

In a situation where there are ten students and each student drank a whole glass of beer on her own, sentence (21) is clearly false. However, if we assume that no students shared any glass of beer between them, the **BC** operator makes sentence (21) true, because there is no relevant set of students with more than five members: all the relevant sets are singletons. For these reasons, in the proposal below we choose to study the counting process of the **N** operator. We leave for further research the empirical study of the exact interpretation of sentences such as (19c), as well as the formal study of the **BC** operator that is motivated by their interpretation.

3.4 Determiner fitting and the witness condition

To overcome the two problems of S&D's mechanism that were pointed out in subsection 3.2, Winter (1998,2001) proposes to reformulate the **N** and **E** operators as one operator called *dfit* (for *determiner fitting*). This operator, unlike S&D's **N** and **E** operators and Dalrymple et al.'s **BC** operator, maps an Atom-Atom determiner into a Set-Set determiner, i.e. a determiner where both arguments can be collective predicates. To define the *dfit* operator, let us first reformulate **N** as an operator from Atom-Atom determiners to Set-Set determiners. This reformulation of the **N** operator is called *count*, and is defined as follows.

Definition 5 (counting operator) *Let D be an Atom-Atom determiner over E. The corresponding Set-Set determiner $count(D)$ is defined for all $\mathcal{A}, \mathcal{B} \subseteq \wp(E)$ by:*

$$(count(D))(\mathcal{A})(\mathcal{B}) \stackrel{def}{=} D(\cup\mathcal{A})(\cup(\mathcal{A} \cap \mathcal{B})).$$

By giving a symmetric Set-Set denotation to collective determiners, this definition involves two separate sub-processes within the process of counting members of collections. The first sub-process is the *intersection* of the right argument with the left argument of the determiner. The second sub-process is the *union* of the sets in each of the two arguments. The intersection sub-process reflects the *conservativity* of (distributive/collective) quantification in natural language: the elements of the right argument that need to be considered are only those that also appear in the left argument. This also holds for the Set-Set determiner $count(D)$.[14] The union sub-process is simply a natural 'participation' adjustment of the type of the Atom-Atom determiner's arguments: for any collective predicate $\mathcal{A}$, an atom x is in $\cup\mathcal{A}$ iff x participates in a set in $\mathcal{A}$.

The *count* operator generalizes S&D's **N** operator in the following sense.

Fact 2 *For every conservative Atom-Atom determiner D over E, for all $A \subseteq E$ and $\mathcal{B} \subseteq \wp(E)$: $(count(D))(\wp(A))(\mathcal{B}) = (\mathbf{N}(D))(A)(\mathcal{B})$.*

Thus, like the **N** operator, *count* respects the semantics of conservative determiners on distributive predicates (cf. fact 1).

Corollary 3 *For every conservative Atom-Atom determiner D over E, for all $A, B \subseteq E$: $(count(D))(\wp(A))(\wp(B)) = D(A)(B)$.*

As with Dalrymple et al.'s **BC** operator, a counting process (of the *count* operator) is combined with an existential requirement. In order to do that, a useful notion is the notion of *witness set* from Barwise and Cooper (1981).

Definition 6 (witness set) *Let D be an Atom-Atom determiner over E and let A and W be subsets of E. We say that W is a* witness set *of D and A iff $W \subseteq A$ and $D(A)(W) = 1$.*

For example, the only witness set of the determiner $\mathbf{every}'$ and the set $\mathbf{man}'$ is the set $\mathbf{man}'$ itself. A witness of $\mathbf{some}'$ and $\mathbf{man}'$ is any non-empty subset of $\mathbf{man}'$. We sometimes sloppily refer to a witness set of a determiner D and a set A as 'witnessing the *quantifier* $D(A)$.'[15]

To the *count* operator we now add an 'existential' condition that is formalized using a *witness operator*.

[14] Note that we still assume that the Atom-Atom determiner D that is lifted by the *count* operator is conservative. However, even when D is conservative, lifting it by an alternative operator $count'(D) = D(\cup\mathcal{A})(\cup\mathcal{B})$, which does away with the intersection process within *count*, would not guarantee sound conservativity equivalences such as between (i) and (ii) below.

(i) All the students are similar.

(ii) All the students are similar students.

Using the $count'$ operator, sentence (i) would be treated, contrary to intuition, as being true if every student is similar to something else (potentially a non-student). But sentence (ii) would be treated by the $count'$ operator as being false in such a situation.

[15] Barwise and Cooper define witness sets on quantifiers explicitly, but they reach the argument A indirectly by defining what they call a *live on set* of the quantifier. This complication is unnecessary for our purposes.

Definition 7 (witness operator) *Let D be an Atom-Atom determiner over E. The corresponding Set-Set determiner $wit(D)$ is defined for all $\mathcal{A}, \mathcal{B} \subseteq \wp(E)$ by:*

$$(wit(D))(\mathcal{A})(\mathcal{B}) = 1 \Leftrightarrow \mathcal{A} \cap \mathcal{B} = \emptyset \vee \exists W \in \mathcal{A} \cap \mathcal{B}[D(\cup\mathcal{A})(W) = 1].$$

In words: the witness operator maps an Atom-Atom determiner D to a Set-Set determiner that holds of any two sets of sets $\mathcal{A}, \mathcal{B}$ iff their intersection $\mathcal{A} \cap \mathcal{B}$ is empty or contains a witness set of D and $\cup\mathcal{A}$.

A similar strategy for quantification over witness sets is proposed in Szabolcsi (1997). While Szabolcsi's witness operation is used only for $\mathrm{MON}\uparrow$ determiners, the witness operator that is defined above is designed to be used for all determiners. This is the reason for the disjunction in the definition of the *wit* operator with an emptiness requirement on $\mathcal{A} \cap \mathcal{B}$. As we shall see below, this will allow us to apply the witness operator as a general strategy, also in cases like (9), without imposing undesired existential requirements as in (10). The general *determiner fitting* operator that we use is simply a conjunction of the counting operator and the witness operator.

Definition 8 (determiner fitting operator) *Let D be an Atom-Atom determiner over E. The corresponding Set-Set determiner $dfit(D)$ is defined for all $\mathcal{A}, \mathcal{B} \subseteq \wp(E)$ by:*

$$(dfit(D))(\mathcal{A})(\mathcal{B}) = 1 \Leftrightarrow (count(D))(\mathcal{A})(\mathcal{B}) = 1 \wedge (wit(D))(\mathcal{A})(\mathcal{B}) = 1.$$

To exemplify the operation of the *dfit* operator, consider the analysis in (23) below of sentence (14), repeated as (22). In this analysis, the noun *students* is treated as the distributed set of atoms $\wp(\mathbf{student'})$. This is needed in order to match the general type of the left argument of the Set-Set determiner that is derived by the *dfit* operator.

(22) Exactly 5 students drank a whole glass of beer together.

(23) $dfit(\mathbf{exactly\_5'})(\wp(\mathbf{student'}))(\mathbf{drink\_beer'})$
$\Leftrightarrow |\{x \in A : A \subseteq \mathbf{student'} \wedge \mathbf{drink\_beer'}(A)\}| = 5$
$\wedge \exists W \subseteq \mathbf{student'}[\mathbf{drink\_beer'}(W) \wedge |W| = 5]$

The first conjunct in this formula is derived by the *count* operation, and guarantees that *exactly* five students participated in sets of students drinking beer. The second conjunct is a result of the witness condition, and it verifies that there exists at least one such set that is constituted by exactly five students.

By combining the counting process and the existential process in this way, the *dfit* operator captures some properties of collective quantification that seem quite puzzling under S&D's double lifting strategy. On the one hand, as Van Benthem's problem indicates, in sentences such as (9) and (12), with $\mathrm{MON}\downarrow$ determiners, the existential strategy is problematic and only the **N** operator is needed. In sentences such as (15), with $\mathrm{MON}\uparrow$ determiners, the existential strategy is needed and the **N** operator is redundant. Moreover, when the determiner is $\mathrm{MON}\!\sim$, as in (14), the existential analysis is needed in combination with the neutral analysis. The dfit operator distinguishes correctly between these cases. As we will presently see, in those cases where a simple existential analysis would be problematic, the witness condition is trivially met due to the counting condition within *dfit*. We characterize two such cases: cases such as (12), where the two arguments of the determiner are distributed sets of atoms, and cases such as (9), where the determiner is downward right-monotone. In other cases, the witness condition does add a non-trivial requirement to the counting operator.

First, let us observe that when the arguments of a Set-Set determiner $\mathit{dfit}(D)$ are two distributed sets of atoms $\wp(A)$ and $\wp(B)$, the witness operator adds nothing to the requirement that $D(A)(B) = 1$.

Fact 4 *Let D be a conservative Atom-Atom determiner over E. Then for all $A, B \subseteq E$: if $D(A)(B) = 1$ then $(\mathit{wit}(D))(\wp(A))(\wp(B)) = 1$.*

Proof. Assume that $D(A)(B) = 1$. By conservativity, the witness set $W = A \cap B$ satisfies the existential requirement in *wit*. □

From this fact and corollary 3 it directly follows that the witness operator is redundant in dfit when the arguments of the determiner are both distributed sets of atoms.

Corollary 5 *Let D be a conservative Atom-Atom determiner over E. Then for all $A, B \subseteq E$: $(\mathit{dfit}(D))(\wp(A))(\wp(B)) = D(A)(B)$.*

Similarly, when a determiner is downward monotone in its right argument, the witness operator is again redundant in the definition of dfit:

Fact 6 *Let D be a $\mathrm{MON}\!\downarrow$ Atom-Atom determiner over E. Then for all $\mathcal{A}, \mathcal{B} \subseteq \wp(E)$: $(\mathit{dfit}(D))(\mathcal{A})(\mathcal{B}) = (\mathit{count}(D))(\mathcal{A})(\mathcal{B})$.*

In other cases — that is, when $\mathcal{A}$ or $\mathcal{B}$ are not DSAs and D is not $\mathrm{MON}\!\uparrow$ — $(\mathit{count}(D))(\mathcal{A})(\mathcal{B})$ does *not* entail $(\mathit{wit}(D))(\mathcal{A})(\mathcal{B})$. Accordingly, an existential requirement *is* invoked by the sentence. This is illustrated by the entailments from the sentences in (24) to sentence (25):

(24) a. More than/exactly five students drank a whole glass of beer together.

b. More than five/exactly five students who drank a whole glass of beer together smiled.

c. More than/exactly five students who drank a whole glass of beer together hit each other later.

(25) ⇒ There was (at least) one group of more than/exactly five students who drank a whole glass of beer together.

These entailments are not captured by the *count* operator alone, but they follow from the *wit* requirement within the *dfit* operator.

3.5 Determiners that are trivial for plurals

Before moving on to the next section and to the monotonicity properties of Set-Set determiners under the *dfit* operator, there is an additional notion that we need to introduce, which refines the right triviality property for plural determiners. An illustration of the point is the behavior of the definite article. In singular sentences such as (26) below, the 'Russellian' interpretation of the definite article in GQT analyses it as a universal determiner that in addition imposes a uniqueness condition on its left argument. The definition of this determiner is given in (27).

(26) The student smiled.

(27) $\mathbf{the}'_{sg}(A)(B) = 1 \;\Leftrightarrow\; A \subseteq B \wedge |A| = 1$

By contrast, in a plural sentence such as (28) below, this definition would be inadequate. The sentence here imposes a *plurality* requirement on the left argument of the determiner, rather than uniqueness. A possible definition of this determiner is given in (29).

(28) The students smiled.

(29) $\mathbf{the}'_{pl}(A)(B) = 1 \;\Leftrightarrow\; A \subseteq B \wedge |A| > 1$

Without getting into the question of definites,[16] let us make one simple point. We do not expect the meaning in (27), which is appropriate for the singular definite article, to be a meaning of any plural determiner. The reason is that such a determiner function, which imposes singularity on its left argument, would contradict the implication, prominent with plurals, that there are at least two elements in the left argument.[17] Consequently, no plural noun could appear with such a determiner without leading to a trivial statement: a contradiction or a tautology. As with singular determiners, we do not expect such trivialities with plural determiners. Crucially, note that the determiner in (27) is not RTRIV (or LTRIV). We can therefore assume that plural determiners show a stronger notion of non-triviality than RTRIV, which we call *triviality for plurals* (PTRIV). Formally:

Definition 9 (triviality for plurals) *A determiner D over E is called* trivial for plurals *(PTRIV) iff for all $A, B, B' \subseteq E$: if $|A| > 1$ then $D(A)(B) = D(A)(B')$.*

[16]Most contemporary works assume that the definite article does not denote a determiner, but some version of the *iota* operator. For works that propose a unified definition of the definite article for singular and plural NPs, see Sharvy (1980) and Link (1983). We believe that this view on articles is justified (cf. Winter (2001)), and that (in)definite articles such as *a* and *the* should not denote determiners in GQT. Therefore, the *dfit* operator does not apply to these articles. For one thing, it does not seem possible to derive the collective reading of the definite article from its singular denotation in (27) using the same operator that applies to other determiners.

[17]Whether this implication is truth-conditional or presuppositional is irrelevant here. See some discussion of this point in Krifka (1992), Schwarzschild (1996), Chierchia (1998) and Winter (1998,2001).

Informally, a PTRIV determiner is indifferent to the identity of its right argument whenever its left argument is a set with two or more entities. The determiner $\mathbf{the}'_{sg}$ as defined above is PTRIV but not RTRIV. We hypothesize that all plural determiner expressions in natural language (though not necessarily all singular determiner expressions) denote non-PTRIV determiners. This hypothesis about plural determiners will play a role in the next section.

4 Monotonicity properties of collective determiners

The facts that were mentioned in the introduction indicate that standard monotonicity properties of determiners are not always preserved when they apply to collective predicates. In this section we show that, using the *count* operator, monotonicity properties in the right argument of a determiner are always preserved, in agreement with intuition. However, whether or not a determiner preserves its left monotonicity property when it applies to collective predicates, depends on its monotonicity property in the *right* argument. We observe that the reason for these different results for determiners of different monotonicity properties is the asymmetric *conservativity* element within the definition of the *count* operator, which intersects the right argument with the left argument, but not vice versa. Further, we mention without proof that the same results concerning (non-)preservation of monotonicity properties hold for the *dfit* operator, which is defined using *count*.

The following fact summarizes all the cases where (non-)monotonicity is preserved under *count*.

Fact 7 *Let D be a determiner over E. If D belongs to one of the classes $\uparrow\mathrm{MON}\uparrow$, $\downarrow\mathrm{MON}\downarrow$, $\mathrm{MON}\uparrow$ or $\mathrm{MON}\downarrow$, then the Set-Set determiner $count(D)$ belongs to the same class. If D is conservative and $\sim\mathrm{MON}$ ($\mathrm{MON}\sim$), then $count(D)$ is also $\sim\mathrm{MON}$ ($\mathrm{MON}\sim$).*

Proof sketch. For the monotone cases the claim immediately follows from the definition of *count*.

For the non-monotone cases it is enough to note that if D is $\sim\mathrm{MON}$, then there are $A'_1 \subseteq A_1 \subseteq E$, $A_2 \subseteq A'_2 \subseteq E$ and $B, C \subseteq E$ s.t. $D(A_1)(B) = 1$, $D(A'_1)(B) = 0$, $D(A_2)(C) = 1$ and $D(A'_2)(C) = 0$. Since D is conservative, we can apply corollary 3 (page 13) and get: $(count(D))(\wp(A_1))(\wp(B)) = 1$, $(count(D))(\wp(A'_1))(\wp(B)) = 0$, $(count(D))(\wp(A_2))(\wp(C)) = 1$ and $(count(D))(\wp(A'_2))(\wp(C)) = 0$, which shows that $count(D)$ is $\sim\mathrm{MON}$ too. The proof is similar when D is $\mathrm{MON}\sim$. □

Fact 7 explains why in many cases, as in the following examples, determiners do not show any surprising monotonicity patterns when they appear with collective predicates.

(30) Some (rich) students drank a whole glass of (dark) beer together $\Rightarrow$ Some students drank a whole glass of beer together. (*some* is $\uparrow\mathrm{MON}\uparrow$)

(31) No students drank a whole glass of beer together $\Rightarrow$ No (rich) students drank a whole glass of (dark) beer together. (*no* is $\downarrow$MON$\downarrow$)

(32) All/most of the students drank a whole glass of dark beer together $\Rightarrow$ All/most of the students drank a whole glass of beer together.
(*all* and *most* are MON$\uparrow$)

(33) Exactly five students drank a whole glass of beer together $\not\Rightarrow$
$\not\Leftarrow$ Exactly five rich students drank a whole glass of (dark) beer together.
(*exactly five* is $\sim$MON$\sim$)

However, as we saw in (4) above, determiners are sometimes more surprising in their monotonicity behavior with collective predicates. To cover all the monotonicity classes of determiners, we also have to consider the left argument of determiners such as *all*, *not all* and *some but not all*, which are monotone in their left argument but have a different monotonicity property in their right argument. The theorem below will show that almost all these determiners lose their left monotonicity under *count*. The only exceptions are PTRIV determiners that are $\downarrow$MON$\uparrow$ or $\uparrow$MON$\downarrow$. Provably, these determiners preserve left monotonicity under *count*. However, as claimed above, PTRIV determiners are not expected in the class of plural determiners in natural language. Moreover, it is not hard to show that in each of the monotonicity classes $\downarrow$MON$\uparrow$ and $\uparrow$MON$\downarrow$ there is only one PTRIV determiner that is conservative, PI and not RTRIV. These two determiners are the following determiners – D^0 and its complement $\neg D^0$:

$$\forall A, B \subseteq E : D^0(A)(B) = 1 \Leftrightarrow A \subseteq B \ \wedge \ |A| \leq 1;$$

$$\forall A, B \subseteq E : (\neg D^0)(A)(B) = 1 \Leftrightarrow D^0(A)(B) = 0.$$

Once these two special cases are observed, we can establish the following result.

Theorem *Let D be a conservative determiner over E that is not RTRIV. If D satisfies one of the conditions (i) and (ii) below, then the Set-Set determiner $count(D)$ is $\sim$MON.*

(i) D is non-PTRIV and is either $\downarrow$MON$\uparrow$ or $\uparrow$MON$\downarrow$.

(ii) D is PI and is either $\downarrow$MON$\sim$ or $\uparrow$MON$\sim$.

To make the proof of this theorem more readable, we first prove the following lemma.

Lemma 8 *Let D be a conservative determiner over E. If D satisfies one of the conditions (i) and (ii) below then there are $X, Y, Y' \subseteq E$ s.t. $|X| > 1$, $Y' \subset Y \subseteq X$, $D(X)(Y) = 1$ and $D(X)(Y') = 0$.*

(i) D is non-PTRIV and $\downarrow$MON$\uparrow$.

(ii) D *is PI and* $\downarrow$MON$\sim$.

Proof. Assume first that D is non-PTRIV and $\downarrow$MON$\uparrow$. Because D is non-PTRIV and conservative, there are $B, B' \subseteq A \subseteq E$ s.t. $|A| > 1$, $D(A)(B) = 1$ and $D(A)(B') = 0$. Since D is MON$\uparrow$, it follows from $D(A)(B) = 1$ that $D(A)(B \cup B') = 1$. Therefore, we can choose $X = A$, $Y = B \cup B'$ and $Y' = B'$.

If, on the other hand, D is PI and $\downarrow$MON$\sim$, then (by $\downarrow$MON$\sim$ and conservativity) there are $B' \subset B \subseteq A \subseteq E$ and $C \subset C' \subseteq A' \subseteq E$ s.t. the following hold:

($*$) $D(A)(B) = 1$ and $D(A)(B') = 0$ (D is not MON$\downarrow$);

($**$) $D(A')(C) = 1$ and $D(A')(C') = 0$ (D is not MON$\uparrow$).

It follows that both A and A' are not empty. If $|A| > 1$ then simply choose $X = A$, $Y = B$ and $Y' = B'$. Otherwise, $A = B = \{x\}$ for some x in E, and $B' = \emptyset$. Let π be a permutation on E s.t. $\pi(x) = y$ for some $y \in A'$. Since D is PI, it follows from ($*$) that $D(\{y\})(\{y\}) = 1$ and $D(\{y\})(\emptyset) = 0$. Clearly, $A' \setminus \{y\} \neq \emptyset$. Otherwise, $A' = C' = \{y\}$, and from ($**$), $D(\{y\})(\{y\}) = 0$. Therefore, $|A'| > 1$. Since D is $\downarrow$MON, it follows from $D(\{y\})(\emptyset) = 0$ that $D(A')(\emptyset) = 0$. Therefore, we can choose $X = A'$, $Y = C$ and $Y' = \emptyset$. □

Proof of theorem. We first prove the theorem for a determiner D that is either $\downarrow$MON$\uparrow$ or $\downarrow$MON$\sim$. Because D is not RTRIV, it cannot be $\uparrow$MON. Using the same reasoning as in the proof of the non-monotone cases in fact 7, it is straightforward to show that in each of the two cases $count(D)$ is not $\uparrow$MON. It is left to be shown that $count(D)$ is also not $\downarrow$MON. By lemma 8, there are $X, Y, Y' \subseteq E$ s.t. $|X| > 1$, $Y' \subset Y \subseteq X$, and the following holds:

($*$) $D(X)(Y) = 1$ and $D(X)(Y') = 0$.

Define three subsets of $\wp(E)$, $\mathcal{A}$, $\mathcal{A}'$ and $\mathcal{B}$ as follows:

$$\mathcal{A} = \wp(X),\ \mathcal{A}' = \wp(X) \setminus \{Y\} \text{ and } \mathcal{B} = \{Y, Y'\}.$$

Since $\mathcal{A}' \subseteq \mathcal{A}$, $\cup\mathcal{A} = \cup\mathcal{A}' = X$, $\cup(\mathcal{A} \cap \mathcal{B}) = Y$ and $\cup(\mathcal{A}' \cap \mathcal{B}) = Y'$, it follows from ($*$) that $(count(D))(\mathcal{A})(\mathcal{B}) = D(\cup\mathcal{A})(\cup(\mathcal{A} \cap \mathcal{B})) = 1$ and $(count(D))(\mathcal{A}')(\mathcal{B}) = D(\cup\mathcal{A}')(\cup(\mathcal{A}' \cap \mathcal{B})) = 0$. Hence, $count(D)$ is not $\downarrow$MON.

Assume now that D is either $\uparrow$MON$\downarrow$ or $\uparrow$MON$\sim$. Again, it is straightforward to show that $count(D)$ is not $\downarrow$MON. To see that $count(D)$ is also not $\uparrow$MON, consider the negation of D: $\neg D$, defined by $(\neg D)(A)(B) = 1 \Leftrightarrow D(A)(B) = 0$. The determiner $\neg D$ is either $\uparrow$MON$\downarrow$ or $\uparrow$MON$\sim$, respectively. Further, $\neg D$ is non-PTRIV if and only if D is non-PTRIV, and the same holds for the properties CONS, PI and non-RTRIV. Thus, it follows from condition ($*$) above that for the same X, Y and Y':

($**$) $(\neg D)(X)(Y) = 0$ and $(\neg D)(X)(Y') = 1$.

Using the same $\mathcal{A}$, $\mathcal{A}'$ and $\mathcal{B}$ as above we get that $count(D)$ is not $\uparrow\mathrm{MON}$. $\square$

In the proof of the theorem we use one construction of subsets of $\wp(E)$ — $\mathcal{A}$, $\mathcal{A}'$ and $\mathcal{B}$ — that applies to all four classes of determiners. However, it is not always convenient to apply this construction to sentences in natural language, because the set $\mathcal{A}'$ is neither DSA nor a purely collective predicate (i.e. it contains singletons in it). To overcome this empirical difficulty, assume that condition ($*$) is satisfied for some $X = Y$ and a determiner D that satisfies the conditions in the theorem. This assumption is tenable, with no loss of generality, for all determiners that are $\downarrow\mathrm{MON}\uparrow$. Under this assumption, we can use the following construction. Leave $\mathcal{A}$ and $\mathcal{B}$ as they are in the proof, and use the following $\mathcal{A}''$ instead of $\mathcal{A}'$:

$$\mathcal{A}'' = \{A \subset X : |X \setminus A| = 1\} \cup \{Y'\}.$$

Now consider the following two sentences:

(34) a. All the students drank a whole glass of beer together.

b. All the students who've been roommates drank a whole glass of beer together.

Assume that $X = Y = \{s_1, s_2, s_3\}$ and that $Y' = \{s_1, s_2\}$. Clearly, these sets satisfy condition ($*$) with respect to the determiner *all*. Following the construction above, $\mathcal{A} = \wp(X)$, $\mathcal{A}'' = \{\{s_1, s_2\}, \{s_1, s_3\}, \{s_2, s_3\}\}$ and $\mathcal{B} = \{\{s_1, s_2, s_3\}, \{s_1, s_2\}\}$. Assume now that the denotation of *students* is $\mathcal{A}$, and that the denotation of *students who've been roommates* is $\mathcal{A}''$, i.e. all the couples of students. Assume further that the denotation of *drank a whole glass of beer together* is $\mathcal{B}$. In this situation it is clear that sentence (34a) is true. However, since, for instance, s_1 and s_3 were roommates, but did not drink a whole glass of beer together, sentence (34b) is not true in this situation.

An example for a $\uparrow\mathrm{MON}\sim$ determiner is the determiner *some but not all*. This determiner is formally defined as follows, for all $A, B \subseteq E$:

$$\mathbf{some\_but\_not\_all}'(A)(B) = 1 \Leftrightarrow A \cap B \neq \emptyset \wedge A \setminus B \neq \emptyset.$$

Consider the following two sentences:

(35) a. Some but not all of the students who've been roommates drank a whole glass of beer together.

b. Some but not all of the students drank a whole glass of beer together.

Clearly, the same X, Y and Y' from the previous example satisfy condition ($**$) with respect to the determiner *some but not all*. Following the same line, assume that the denotations of *students*, *students who've been roommates* and *drank a whole glass of beer together* are the same as in the previous example. Now, sentence (35a) is true in this situation, since there is a set of students that were roommates and also drank a whole glass of beer together, namely $\{s_1, s_2\}$, but it is not true that *all* the students who've been roommates drank a whole glass of beer

Monotonicity of D	Monotonicity of $count(D)$	Example
$\uparrow$MON$\uparrow$	$\uparrow$MON$\uparrow$	*some*
$\downarrow$MON$\downarrow$	$\downarrow$MON$\downarrow$	*less than five*
$\downarrow$MON$\uparrow$	$\sim$MON$\uparrow$ (!)	*all*
$\uparrow$MON$\downarrow$	$\sim$MON$\downarrow$ (!)	*not all*
$\sim$MON$\sim$	$\sim$MON$\sim$	*exactly five*
$\sim$MON$\downarrow$	$\sim$MON$\downarrow$	*not all and (in fact) less than five (of the)*
$\sim$MON$\uparrow$	$\sim$MON$\uparrow$	*most*
$\downarrow$MON$\sim$	$\sim$MON$\sim$ (!)	*all or less than five (of the)*
$\uparrow$MON$\sim$	$\sim$MON$\sim$ (!)	*some but not all (of the)*

Table 2: (non-)monotonicity under $count$

together (cf. the argument in the previous example). On the other hand, sentence (35b) is not true in this situation, simply because all the students drank a whole glass of beer together.

Table 2 summarizes the (non-)preservation of monotonicity properties under *count* for the nine classes of determiners according to the monotonicity of their two arguments. Note again that for each of the two classes $\downarrow$MON$\uparrow$ and $\uparrow$MON$\downarrow$, there is an exception to the result that is mentioned in the table: the PTRIV determiners. The exclamation marks emphasize the cases in which left monotonicity is not preserved.

There are two natural extensions to these results, which we state here without proof. First, we note that fact 7 and the 'monotonicity loss' theorem equally hold when *count* is replaced by *dfit*. Thus, adding the witness condition to *count* does not change the monotonicity (non-)preservation results that were established above for *count*. The proof of this claim is quite laborious, but routine. Linguistically, it implies that existential processes in collective quantification should not lead to problems that are similar to Van Benthem's problem, or to any other change in the monotonicity properties of determiners beyond what was shown above.

Another point that worths mentioning is that the results that were shown above equally hold for *global determiners*: functors from domains E to determiners D_E over E. It is important to appeal to global functors because linguistic items such as *all*, *some*, *five* etc. have global definitions and properties, and are not simply defined over a given domain, as assumed throughout this article. We say that a global determiner D is conservative, permutation invariant, (left/right) trivial or (upward/downward left/right) monotone, if the local determiner D_E satisfies the respective property for any non-empty domain E. Using this global perspective, all the results that were proven above for local determiners equally hold of global determiners that satisfy the *extension* property (cf. Van Benthem (1984)).[18] The

[18]When we say that a global determiner D satisfies extension, this means that given a domain E and any two sets $A, B \subseteq E$, the local determiners $D_{E'}$ s.t. $E \subseteq E'$ all agree on the truth value that they assign to A and B.

reason for assuming this global property is our results concerning non-monotone determiners. Note that a global determiner D is $\sim$MON (MON$\sim$) if and only if there is E s.t. D_E is not $\downarrow$MON (MON$\downarrow$) and there is E' s.t. $D_{E'}$ is not $\uparrow$MON (MON$\uparrow$). From this it does not yet automatically follow that there is a domain E s.t. D_E is $\sim$MON (MON$\sim$), as required in fact 7 and the main theorem. However, provided that a global determiner D is non-trivial in both its arguments and satisfies extension, the existence of such a domain does follow from the assumption that D is $\sim$MON (MON$\sim$). Since most works on GQT assume that natural language determiners satisfy extension (see Keenan and Westerståhl (1996)), the generalization of our results to global determiners is both linguistically and technically straightforward.

5 Conclusion

The formal study of the interactions between quantifiers and collective predicates has to deal with many seemingly conflicting pieces of evidence that threaten to blur the interesting logical questions that these phenomena raise. In this article we have studied the monotonicity properties of collective quantification, which is a central aspect of the problem of collectivity. We showed that to a large extent, the principles that underly monotonicity of collective quantification follow from standard assumptions on quantification in natural language in general. The *count* operator, which is a straightforward extension of Scha's 'neutral' analysis of collective determiners, involves a simple 'conservativity element' – intersection of the right argument with the left argument, and a 'participation element' – union of both set of sets arguments. The conservativity element within the *count* operator is responsible for the two *a priori* unexpected asymmetries in the monotonicity behavior of collective determiners:

1. Only determiners with 'mixed' monotonicity properties change their behavior when they quantify over collections.
2. Only the *left* monotonicity properties of such determiners may change in these cases.

We believe that the reduction of certain asymmetries in the domain of collective quantification to the asymmetric conservativity principle is a desirable result that reveals another aspect of the central role that this principle plays in natural language semantics.

Two open questions should be mentioned. First, in this article we did not address the 'universal' reading that certain collective determiners show, as treated by Dalrymple et al.'s bounded composition operator. More empirical research is needed into these phenomena, which indicate that there may be more than one strategy of plural quantification. The formal properties of such universal strategies and the linguistic restrictions on their application should be further explored. Second, although we characterized the logical monotonicity properties of collective determiners, we did not study the linguistic implications that these properties may have for the analysis of *negative polarity items*. These items normally appear only in downward entailing environments, and it should be checked whether they are sensitive to 'monotonicity loss' under collectivity of *all*. For instance, a sentence such as the following, where *all* is not left downward monotone, is expected not to license the negative polarity item *any* in its left argument.

(36) ?All the students who had any time drank a whole glass of beer together.

Whether or not this expectation is borne out is not clear to us, and we must leave these and other implications of 'monotonicity loss' to further research.

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