MODAL STRUCTURES IN GROUPS AND VECTOR SPACES

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Comment to the Posted Version, 19 March 2023

The most recent posted version of this paper contains a number of typographical corrections communicated to the authors by Peter van Emde Boas, as well as a more substantial addition to Example 2.6 on modally definable subsets of $\mathbb{Z} \times \mathbb{Z}$. There is also an important correction to the formulation of Theorem 4.8, triggered by a question of Yanjing Wang.

In addition, we would like to thank the in-person and on-line audience of the joint LLAMA-LIRA seminar at the ILLC, University of Amsterdam, 9 March 2023, for their many responses and suggestions. We list a few selected topics that were raised, as exploring these might help in exploring the range as well as the limitations of our approach:

- Defining special kinds of subgroups, such as normal subgroups, in our basic modal language extended with the closure modality
- Finding modal perspectives on classical decomposition theorems for groups in terms of irreducible subgroups
- Possible transfer of existing non-finite axiomatizability and undecidabbility results in the theory of group relation algebras in algebraic logic
- Exploring connections to group and semigroup semantics for categorial logics
- Alternative modal-style logizations of group structure, e.g., through the Cayley graph
- Ways of modifying our modal completeness proofs to achieve more control over the vector spaces obtained by our term quotient construction
- Investigating the enforcable dimensions of vector spaces contained in our Henkin models [in our basic modal language, we can enforce infinite dimension – with the modality for linear multiples added, we can also enforce specific finite dimensions]
- Introducing analogues to the inverse structure of vector addition as represented in our logics in current information-based semantics
- Extending our dynamic-logic inspired analysis from groups to the more general setting of group actions on arbitrary models.